An overview of the modelling of fracture by gradient damage models

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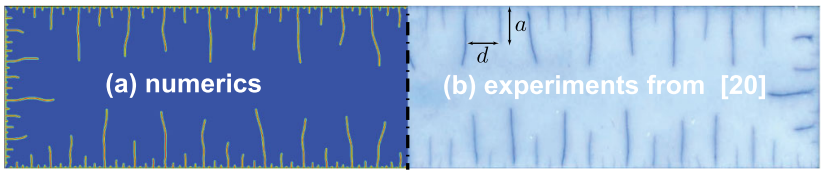
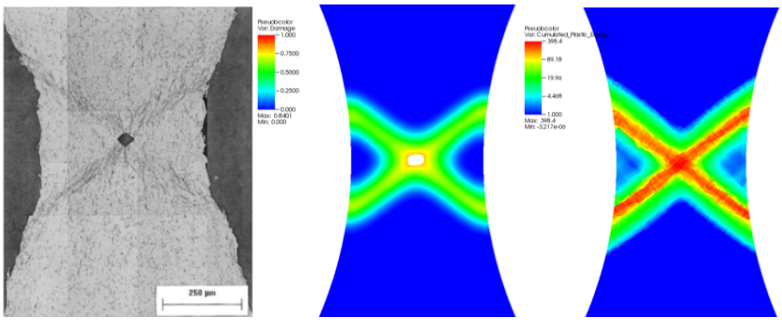
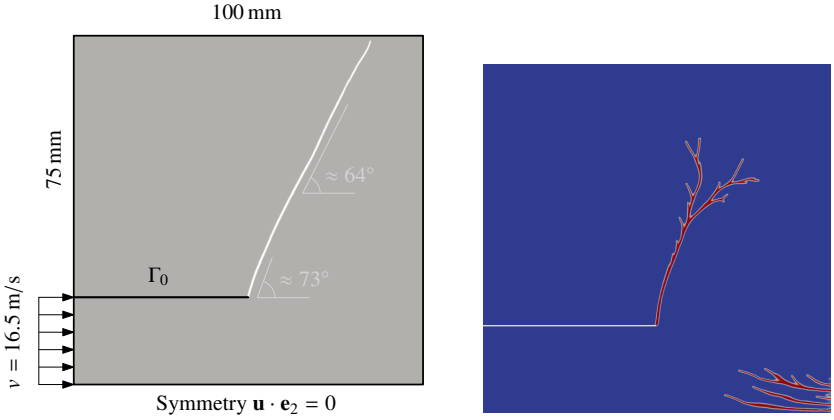
The lecture is devoted to gradient damage models which allow us to describe all the process of degradation of a body including the nucleation of cracks and their propagation. The construction of such model follows the variational approach to fracture [2] and proceeds into two stages: (1) definition of the energy; (2) formulation of the damage evolution problem. The total energy of the body is defined in terms of the state variables which are the displacement field and the damage field in the case of quasi-brittle materials [5], whereas they contain also the plastic strain field in the case of ductile materials [1]. That energy contains in particular gradient damage terms in order to avoid too strong damage localizations. The formulation of the damage evolution problem is then based on the concepts of irreversibility, stability and energy balance, as well in quasi-static as in dynamic [4]. That allows us to construct homogeneous as well as localized damage solutions in a closed form and to illustrate the concepts of loss of stability, of scale effects, of damage localization, and of structural failure. Moreover, the variational formulation leads to a natural numerical method based on an alternate minimization algorithm. Several numerical examples will illustrate the ability of this approach to account for all the process of fracture including a 3D thermal shock problem where the crack evolution is very complex [3].



Numerical simulation of a ceramic slab submitted to a thermal shock by a gradient damage model (a) Computed damage field *d* (blue, d=0; red, d=1). (b) Experimental results from [20]: Y. Shao, Y. Zhang, X. Xu, Z. Zhou, W. Li, and B. Liu, *J. Am. Ceram. Soc.*, 94: 2804 , 2011.



Simulation of the (quasi-static) failure of a 2D-slanted specimen under uniaxial traction by a gradient damage model coupled with plasticity
(left: experiment; center: damage field; right: cumulated plastic strain field)



Simulation by a gradient damage model (without plasticity) of the dynamical propagation of a crack in an impact test (Kalthoff-Winkler experiment)

References

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