Granulation of wet granular media in rotating drum

T.T. Vo¹,², S. Nezamabadi¹,³, P. Mutabaruka¹, J.-Y. Delenne³, F. Radjai¹

¹ LMGC, CNRS, Université de Montpellier; {thanh-trung.vo, franck.radjai, saeid.nezamabadi, patrick.mutabaruka}@umontpellier.fr
² Bridge and Road Department, Danang Architecture University, 553000 Da Nang, Vietnam
³ IATE, CIRAD, INRA, Montpellier SupAgro, Université de Montpellier; jean-yves.delenne@inra.fr

Résumé — We study the agglomeration process of wet granular materials in a rotating drum by using discrete-element simulations. The particle interactions account for the cohesive and viscous forces of the binding liquid. We assume that the liquid is transported by wet particles and keeps a constant volume during granulation. A single granule introduced into the rotating drum grows by capturing wet particles. We find that the granule size increases exponentially with the number of drum rotations, and we investigate the effects of various parameters on the growth process.

Mots clés — Granular matter; Granulation; Capillary bond; Discrete Element Method; Rotating drum.

1 Introduction

The agglomeration of solid particles is a fundamental process in industrial applications such as the manufacture of pharmaceuticals, fertilisers, powder metallurgy and iron-making. Wet agglomeration occurs as a result of the collisions of moist particles in a flowing granular bed. The particle size increase modifies the density and strength of a granular material, and it may improve its flow properties, reduce the segregation of different types of particles, and increase the permeability [1]. The particles are wetted by mixing raw particles with the binding liquid [2]. The nuclei of granules are formed due to the collisional and frictional or capillary and viscous interactions between wet primary particles and by the incorporation of the binding liquid [3].

The size, strength, and texture of granules depend on the process and material parameters. The process parameters include the amount of liquid volume, the size ratio between the drum and mean particle diameter, the rotation speed, the inclination angle, and the filling rate [8, 7, 6]. The material parameters are the binding liquid and raw material properties including the liquid viscosity, particle size distribution, mean particle size and friction coefficient of particles [2]. The agglomeration of wet particles involves three elementary processes: wetting and nucleation, accretion and growth, and erosion (or attrition) and breakage [9]. A granule grows if the accretion rate is larger than the erosion rate. The growth will eventually stop if the liquid volume is finite (no liquid supplied). In continuous supply of liquid, the granule growth can continue until all particles are wetted and as long as the liquid is in the funicular state (no slurry).

In this paper, we use the Discrete Element Method (DEM) to simulate the evolution of a single small granule initially introduced into a rotating drum. The primary particles in the drum are either wet or dry. The wet particles interact through a capillary-viscous force law, and their number represents the amount of liquid inside the drum. We analyze the effects of various material parameters on the evolution of the granule.

2 Numerical method and granulation procedure

2.1 Numerical method

We use a Molecular Dynamics (MD) approach for DEM simulations of the agglomeration process. The motion of each solid spherical particle \( i \) of radius \( R_i \) is governed by Newton’s second law. The particles interact through five different forces: the normal contact force \( f_n \), the tangential friction force
where \( m_i \), \( I_i \), \( r_i \), \( \omega_i \) and \( \mathbf{g} \) denote the mass, the inertia matrix, and the position, rotation and gravity acceleration vectors of particle \( i \), respectively. \( \mathbf{n}^{ij} \) is the normal vector of the contact plane between the particles \( i \) and \( j \), \( \mathbf{t}^{ij} \) denotes the tangential vector belonging to the contact plane \( ij \) and pointing in the direction opposite to the relative displacement of the two particles and \( \mathbf{c}^{ij} \) is the vector pointing from the center of particle \( i \) to the contact point with particle \( j \).

The normal contact force \( f_n \) involves two components \([13]\):

\[
f_n = f_n^e + f_n^d. \tag{2}
\]

where \( f_n^e = k_n \delta_n \) is the normal repulsive elastic force as a linear function of the normal elastic deflection \( \delta_n \) (overlap between two particles), where \( k_n \) is the normal stiffness, and \( f_n^d = \gamma_n \delta_n \) is the normal damping force, proportional to the relative normal velocity \( \dot{\delta}_n \), with \( \gamma_n \) as the normal viscous damping. These forces occur only when there is an overlap, i.e. for \( \delta_n < 0 \). The tangential contact force \( f_t \) is the sum of an elastic force \( f_t^e = k_t \delta_t \) and a damping force \( f_t^d = \gamma_t \delta_t \), where \( k_t \) denotes the tangential stiffness, \( \gamma_t \) is the tangential viscous damping parameter, and \( \delta_t \) and \( \delta_t \) are the tangential displacement and velocity, respectively. The tangential force is bounded by a force threshold \( \mu f_n \) according to the Coulomb friction law, where \( \mu \) is the friction coefficient:

\[
f_t = -\min \left\{ (k_t \delta_t + \gamma_t \delta_t) \mu f_n \right\}. \tag{3}
\]

The capillary force \( f_c \), between two wet particles depends on the liquid volume \( V_b \) of the capillary bridge, liquid-vapor surface tension \( \gamma_l \) and particle-liquid-gas contact angle \( \theta \) \([14]\); see Fig. 1. The capillary force is determined by integrating the Laplace-Young equations. In the pendular state, an approximate solution is given by the expression \([14, 12, 15]\):

\[
f_c = \begin{cases} 
-\kappa R, & \text{for } \delta_n < 0, \\
-\kappa R e^{-\delta_n/\lambda}, & \text{for } 0 \leq \delta_n \leq d_{\text{rupt}}, \\
0, & \text{for } \delta_n > d_{\text{rupt}},
\end{cases} \tag{4}
\]

where \( R = \sqrt{R_i R_j} \) is the geometrical mean radius of two particles of radii \( R_i \) and \( R_j \) and the capillary force pre-factor \( \kappa \) is

\[
\kappa = 2\pi \gamma_l \cos \theta. \tag{5}
\]
This force exists up to a rupture distance \( d_{rupt} \) given by

\[
d_{rupt} = \left(1 + \frac{\theta}{2}\right) V_b^{1/3}.
\]  

The characteristic length \( \lambda \) is the factor that denotes the exponential falloff of the capillary attraction force in equation (4):

\[
\lambda = c \ h(r) \left(\frac{V_b}{R'}\right)^{1/2},
\]

where \( R' = 2R_iR_j/(R_i + R_j) \) and \( r = \max\{R_i/R_j; R_j/R_i\} \) are the harmonic mean radius and the size ratio between two particles. The expression (4) nicely fits the capillary force as obtained from direct integration of the Laplace-Young equation by setting \( h(r) = r^{-1/2} \) and \( c \simeq 0.9 \) [16].

The normal viscous force \( f_{vis} \) is due to the lubrication effect of liquid bonds between particles. Its expression for two spherical particles is [10, 15]:

\[
f_{vis} = \frac{3}{2} \pi R^2 \eta \frac{V_n}{\delta_n},
\]

where \( \eta \) is the liquid viscosity and \( v_n \) denotes the relative normal velocity assumed to be positive when the gap \( \delta_n \) is decreasing. This expression implies that the viscous force diverges when the gap \( \delta_n \) tends to zero. We also introduce a characteristic length \( \delta_{n0} \) representing the size of asperities and assume that the lubrication force is given by:

\[
f_{vis} = \frac{3}{2} \pi R^2 \eta \frac{V_n}{\delta_n + \delta_{n0}} \quad \text{for} \quad \delta_n > 0.
\]

In the case \( \delta_n > 0 \), i.e. for a positive gap, the singularity will not occur as long as there is no contact. When contact occurs, i.e. for \( \delta_n < 0 \), we assume that the viscous force depends only on the characteristic length:

\[
f_{vis} = \frac{3}{2} \pi R^2 \eta \frac{V_n}{\delta_{n0}} \quad \text{for} \quad \delta_n \leq 0.
\]

Here, we keep \( \delta_{n0} = 5 \times 10^{-4} d_{min} \). This value is small enough to allow the normal viscous forces to be effective without leading to its divergence at contact.

### 2.2 Granulation procedure

Figure 2(a) shows the 3D numerical model of a rotating drum defined as cylinder of length \( L \) and diameter \( d_c \) composed geometrically by the juxtaposition of polyhedral elements. This drum is closed by two end planes and it can rotate around \( y \) axis at a given rotation speed \( \omega \). In our simulations, the gravity is perpendicular to the rotation axis.

\[ \text{FIGURE 2 – (a) The geometry of the numerical drum, (b) initial distribution of dry (in gray) and wet (in black) particles with a single granule defined in the center top of the granular bed and 200 wet particles randomly distributed inside the bed.} \]

The samples are composed of 5000 spherical particles placed inside a horizontal drum. All granular beds have the same filling level of \( f = 20\% \) for different values of the size ratio. The size polydispersity of the particles is an important parameter defined by the ratio \( \alpha = d_{max}/d_{min} \) between the largest and
### TABLE 1 – Simulation parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum particle diameter</td>
<td>$d_{\text{min}}$</td>
<td>10</td>
<td>$\mu$m</td>
</tr>
<tr>
<td>Density of particles</td>
<td>$\rho$</td>
<td>3500</td>
<td>kg.m$^{-3}$</td>
</tr>
<tr>
<td>Size ratios</td>
<td>$\alpha$</td>
<td>[1,5]</td>
<td></td>
</tr>
<tr>
<td>Number of particles</td>
<td>$N_p$</td>
<td>5000</td>
<td></td>
</tr>
<tr>
<td>Filling level</td>
<td>$f$</td>
<td>20</td>
<td>$%$</td>
</tr>
<tr>
<td>Friction coefficient</td>
<td>$\mu$</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>Normal stiffness</td>
<td>$k_n$</td>
<td>100</td>
<td>N/m</td>
</tr>
<tr>
<td>Tangential stiffness</td>
<td>$k_t$</td>
<td>80</td>
<td>N/m</td>
</tr>
<tr>
<td>Normal damping</td>
<td>$\gamma_n$</td>
<td>5.10$^{-5}$</td>
<td></td>
</tr>
<tr>
<td>Tangential damping</td>
<td>$\gamma_t$</td>
<td>5.10$^{-5}$</td>
<td></td>
</tr>
<tr>
<td>Surface tension</td>
<td>$\gamma_s$</td>
<td>0.072</td>
<td>N/m</td>
</tr>
<tr>
<td>Contact angle</td>
<td>$\theta$</td>
<td>30.0</td>
<td>degree</td>
</tr>
<tr>
<td>Liquid viscosity</td>
<td>$\eta$</td>
<td>1.0</td>
<td>mPa.s</td>
</tr>
<tr>
<td>Time step</td>
<td>$\delta t$</td>
<td>10$^{-7}$</td>
<td>s</td>
</tr>
</tbody>
</table>

Smallest particles. We considered three different size classes in a range $[d_{\text{min}}, d_{\text{max}}]$. Each size class has the same total volume so that the size distribution is uniform in terms of particle volumes. This distribution generally corresponds to a high packing fraction since small particles fill the pore space between large particles. In our simulations, the size of smallest particles $d_{\text{min}}$ is considered to be constant whereas the size of largest particles $d_{\text{max}}$ varies. All values of the system parameters used in the simulations are listed in Table 1.

A single granule was defined by 100 wet particles inside a spherical probe located at the center top of the granular bed. 200 more wet particles were randomly distributed inside the drum. All other particles are considered to dry. The liquid content of wet particles is set to be $w = V_l / V_g = 0.09$, where $V_l$ denotes the liquid volume in the system and $V_g$ is the wet particle volume. We also assume that there is no excess liquid so that a wet particle cannot form a liquid bridge with a dry particle. The granule size evolves by the accretion or erosion of wet particles. Fig. 2(b) shows the initial distribution of the granular bed together with the initial single granule and free wet particles randomly distributed when a steady flow state is reached by continuous rotation of the drum. As illustrated in Fig. 3, the wet particles represent actually micro-aggregates that are formed by the capillary action of small amounts of liquid. These micro-aggregates can transport the liquid during their motion and they can share their liquids upon collision to form larger aggregates.

\[ Fr = \frac{\omega^2 d_c}{2g}, \]  

\[ \text{FIGURE 3 – (a) Schematic representation of the granulation model, (b) accretion and erosion phenomena of granule in the agglomeration process.} \]
Here, we set $Fr = 0.5$ in all simulations. This corresponds to a flow regime intermediate between the rolling regime and and cascading regime ($0.1 < Fr < 1$).

## 3 Granule growth

![Figure 4](image.png)

**Figure 4** – Evolution of the granule size $N_g$ (in number of particles) as a function of the number of drum rotations $N_r$ for different values of size ratio $\alpha$.

Figure 4 shows an almost exponentially increase of granule size, corresponding to the number $N_g$ of wet particles inside agglomerate, as a function of the number $N_r$ of drum rotations. Here, $d_{\text{min}}$ is constant and equal to 10 $\mu$m. Hence, $d_{\text{max}} = \alpha d_{\text{min}}$ is variable depending on the size ratio $\alpha$. The friction coefficient is set to $\mu = 0.5$. We observe that, except the mono-spheres case ($\alpha = 1$), for which the granule growth is a nearly linear function of the number of drum rotations, the total number of particles in single granule grows exponentially from the initial stage (100 wet spherical particles) to about its double after 50 drum rotations. The rate of the granule growth is proportional to the size ratio $\alpha$ and reaches the steady state after about 30 drum rotations. The exponential increase reflects the gradual decrease of the number of free micro-aggregates inside the granular bed.

![Figure 5](image.png)

**Figure 5** – Total number of the accreted particles $N_g^+$ and eroded particles $N_g^-$ after 50 drum rotations as a function of the size ratio $\alpha$.

The granule growth reflects the accretion and erosion phenomena during the granulation process. Figs. 5(a) and (b) show the total numbers of the accreted particles $N_g^+$ and the eroded particles $N_g^-$ for different values of the size ratio $\alpha$ after 50 drum rotations. They increase quasi-linearly as a function of $\alpha$. It can be explained by the increase of the normal and tangential forces between particles inside the granular flow as a result of the increase of $\alpha$ (or $d_{\text{max}}$) with a constant value of $d_{\text{min}}$. 

4 Conclusions

In this paper, we investigated the granulation process of wet granular materials in a rotating drum by using a three-dimensional molecular dynamics simulations. The numerical method accounts for the cohesive and viscous effects of a small amount of a binding liquid added to the particles. We studied the growth of a single granule placed inside the granular bed. The granule size grows exponentially with the number of the drum rotations in the granular bed as a consequence of the gradual capture of wet particles by the initial single granule. The erosion rate increases as a linear function of the number of drum rotations whereas the cumulative number of accreted particles increases as an almost exponential function of the granulation time for the different values of polydispersity. The granule size growth slows down after several drum rotations as a result of the finite number of wet particles.

Acknowledgments

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Références