

A Double Scale Analysis on the Instability Phenomena of Long Fiber Reinforced Materials

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Abstract — This work proposes a double scale analysis on the instability phenomena of long fiber reinforced composites. The *multilevel finite element method* (FE²) is adopted to realize the real-time interaction between the microscopic and the macroscopic levels. To simulate the fiber buckling, an efficient Fourier-related representative volume element model is proposed, where all fast varying unknowns are replaced by slowly varying unknowns. The nonlinear multiscale system is solved by *Asymptotic Numerical Method* (ANM), which is more reliable and less time consuming than other classical iterative methods.

Keywords — long fiber reinforced composites, instabilities, Fourier series, multilevel finite element method, asymptotic numerical method.

1 Introduction

Composite materials containing slender fiber inclusions widely exist in nature and are also widely used in aviation, aerospace, biology, medicine and other engineering fields. The macroscopic instability of long fiber composites under compressive stress is quite complicated because it not only depends on the macroscopic geometry and load form but also depends on the volume ratio of the fiber/matrix, the matrix's shear modulus and fiber instability wavelength on the microscopic scale. Thus, the instability of long fiber composite structures is a typical multiscale mechanical behavior.

The multiscale modeling for the instability of long fiber composite structures could be traced back to the end of the 20th century [1, 2, 3]. Nezamabadi et al. [4] analyzed the relationship between macroscopic and microscopic instability using the *multilevel finite element method* [5, 6, 7, 8, 9] (FE²) based on the computational homogenization theory. The FE² method realizes the bi-directional real-time transmission of macroscopic and microscopic information by "the macroscopic level transmits the strain to the microscopic *representative volume element* (RVE), and RVE transmits the stress to the macroscopic level". However, this method requires a RVE at each integration point of the macroscopic finite element model, of which the element size depends on the wavelength of the microscopic buckled fiber, resulting in a large amount of computations. Besides, the relationship between the microscopic fiber buckling modes and macroscopic instability is also to be further studied. Thus, the numerical homogenization model still needs to be improved to accurately and efficiently simulate the double-scale instability of long fiber reinforced composites.

In recent years, the Fourier-related envelope model shows high efficiency in solving fiber instability problems, because its element size is independent of the fiber wavelength. Also, it can control the instability modes by a given wavelength, see [10, 11, 12]. Therefore, it is adopted in this work to simulate the fiber buckling. Compared to the traditional Newton-Raphson method, *Asymptotic Numerical Method* (ANM) [13, 14, 4] is more efficient and more stable when applied to strong nonlinear and multi-bifurcation problems. In this method, the unknowns are expanded by high order power series to convert the nonlinear equations into a series of linear equations. The adopted ANM shows high computational efficiency because the tangent matrix is consistent in a nonlinear increment step and its length is self-adaptive. Thus, ANM is used to improve the controllability of the nonlinear path.

General speaking, this work aims to develop a double scale framework with an efficient RVE model for analyzing the instability of long fiber composite structures, where ANM is adopted as the nonlinear solver. The formulation of the proposed multiscale model is given in Section 2. The accuracy and

efficiency of this model are validated by a fully meshed model.

2 Modelisation

The multiscale analysis for long fiber composites is based on the multilevel finite element method, see Figure 1. Specifically, after solving the microscopic problem with a fine meshed finite element model, the effective tangent matrix $\bar{\mathbb{L}}$ and first Piola-Kirchhoff stress $\bar{\mathbf{P}}$ of a macroscopic point are transferred from its corresponding RVE. Meanwhile, the displacement gradient $\bar{\boldsymbol{\theta}}$ at the macroscopic point is input to the corresponding RVE.

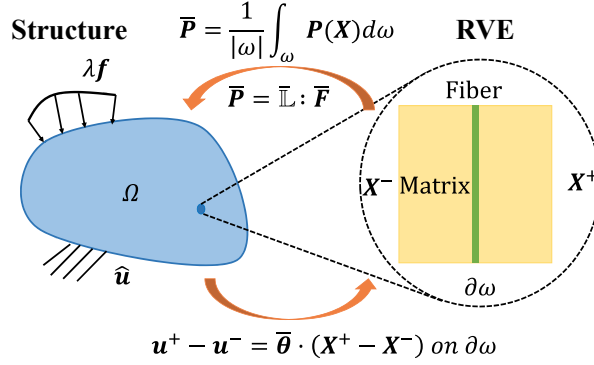


Figure 1: Basic concept of FE².

The macroscopic problem is formulated as:

$$\begin{cases} \int_{\Omega} \bar{\mathbf{P}} : \delta \bar{\mathbf{F}} d\Omega = \lambda \int_{\partial\Omega_{\sigma}} \mathbf{f} \cdot \delta \bar{\mathbf{u}} d\Lambda \\ \bar{\mathbf{F}} = \nabla_{\bar{\mathbf{x}}} \bar{\mathbf{u}} + \mathbf{I} \end{cases} \quad (1)$$

where $\bar{\mathbf{u}}$ represents the macroscopic displacement field, prescribed load \mathbf{f} the external force applied on boundary $\partial\Omega_{\sigma}$ and its intensity λ . Macroscopic deformation gradient tensor $\bar{\mathbf{F}}$ is associated with the work-conjugate stress $\bar{\mathbf{P}}$, \mathbf{I} the second-order identify tensor.

As the constitutive relationship between $\bar{\mathbf{P}}$ and $\bar{\mathbf{F}}$ is unknown, $\bar{\mathbf{P}}$ is to be transferred from the microscopic scale by solving nonlinear equations at microscopic level using finite element method. Subsequently, the microscopic problem is formulated:

$$\begin{cases} \int_{\omega} \mathbf{P} : \nabla_{\mathbf{x}} \delta \mathbf{u} d\omega = \int_{\omega} \mathbf{S} : \delta \boldsymbol{\gamma} d\omega = 0, \\ \mathbf{P} = \mathbf{F} \cdot \mathbf{S}, \\ \boldsymbol{\gamma} = \frac{1}{2} (\mathbf{F}^T \cdot \mathbf{F} - \mathbf{I}), \\ \mathbf{S}(\mathbf{X}) = \mathbb{C}^{(r)} : \boldsymbol{\gamma}(\mathbf{X}), \\ \mathbf{u} = \bar{\boldsymbol{\theta}} \cdot \mathbf{X} \text{ or } \mathbf{u}^+ - \mathbf{u}^- = \bar{\boldsymbol{\theta}} \cdot (\mathbf{X}^+ - \mathbf{X}^-) \text{ on } \partial\omega. \end{cases} \quad (2)$$

where \mathbf{P} is the first Piola-Kirchhoff stress at microscopic point \mathbf{X} , \mathbf{u} the microscopic displacement, $\bar{\boldsymbol{\theta}} = \nabla_{\bar{\mathbf{x}}} \bar{\mathbf{u}}$ the macroscopic displacement gradient, \mathbf{S} the second Piola-Kirchhoff stress, \mathbb{C} the fourth-order elastic tensor associated with phase (r) , $\boldsymbol{\gamma}$ Green-Lagrange strain tensor, the superscripts '+' and '-' specify the nodes on opposite boundaries, the domain of RVE and its boundary denote by ω and $\partial\omega$ respectively.

The tangent constitutive matrix $\bar{\mathbb{L}}$ for each integration point of macroscopic model is computed according to $\bar{\mathbb{L}} = \partial \bar{\mathbf{P}} / \partial \bar{\mathbf{F}}$, where the macroscopic stress $\bar{\mathbf{P}}$ is obtained by taking the average value of microscopic stress \mathbf{P} over RVE: $\bar{\mathbf{P}} = 1/|\omega| \int_{\omega} \mathbf{P} d\omega$. For more details, please refer to Xu et al. [8].

Adopting Fourier series [10, 11, 12], displacement \mathbf{u} is expanded as:

$$\mathbf{u} = \mathbf{u}_0 + (\mathbf{u}_R + i\mathbf{u}_I) e^{iqx} + (\mathbf{u}_R - i\mathbf{u}_I) e^{-iqx}. \quad (3)$$

where $\mathbf{u}_R = \mathbf{u}_1 \cos(\varphi)$, $\mathbf{u}_I = \mathbf{u}_1 \sin(\varphi)$. Insertion of Eq. (3) in Eq. (2) yields the microscopic formulation in the Fourier-related form.

The multiscale system is solved by ANM [13, 14, 12], which has high efficiency in strong nonlinear problems compared with classical interactive procedures, such as Newton-Raphson method. By expanding unknowns Ψ and load parameter λ into power series with respect to a path parameter a , a nonlinear system can be transferred into a series of linear problems:

$$\begin{Bmatrix} \Psi(a) \\ \lambda(a) \end{Bmatrix} = \begin{Bmatrix} \Psi_0 \\ \lambda_0 \end{Bmatrix} + \sum_{p=1}^N a^p \begin{Bmatrix} \Psi_p \\ \lambda_p \end{Bmatrix}, \quad p \in [1, N] \quad (4)$$

where $(\cdot)(a)$ refers to the quantities defined continuously with respect to path parameter a , $(\cdot)_0$ denotes an initial solution such that $(\cdot)_0 = (\cdot)(0)$, N is the truncation order of power series, and $(\cdot)_p$ indicates a term to be determined at order p . For more details, please refer to Refs. [7, 9].

3 Numerical results

The proposed multiscale Fourier-series based model is validated by comparison with a fully meshed model on ABAQUS. A fiber reinforced composite plate subjected to compressive loads is examined, see Figure 2a. A displacement $\lambda \hat{\mathbf{u}}$ is applied on the top edge of the plate along the fiber's length direction. The left and the right edges are fixed in the horizontal direction, while the bottom edge is fixed in the vertical direction. Figure 2b gives the buckling mode of the fibers in the fully meshed ABAQUS model solved by Newton-Raphson method. A total number of 122,880 4-node bilinear plane stress quadrilateral reduced integration elements are used in this fully meshed model. In this example, the volume fraction of the fiber is 1/20. Both the fiber and the matrix are assumed isotropic and linear elastic. Their detailed material properties are given in Table 1.

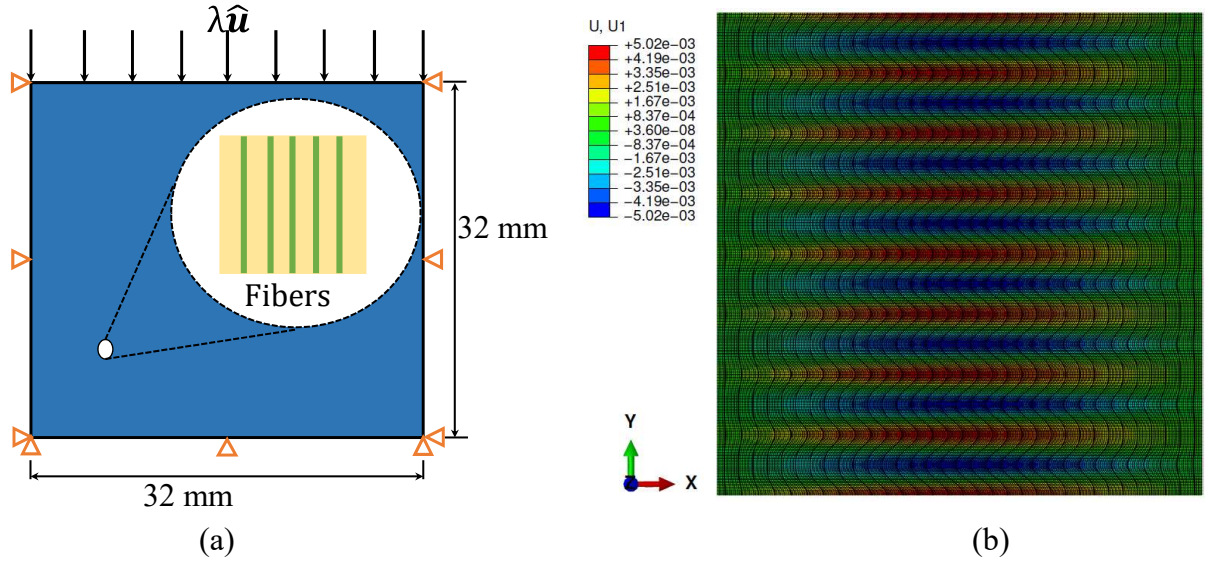


Figure 2: (a) the fiber reinforced composite plate subjected to compressive displacement load; (b) the transversal displacement (mm) distribution of the fully meshed model, deformation scale=100.

Table 1: Material parameters for matrix and fiber.

E_m (MPa)	E_f (MPa)	ν_m	ν_f
100	100000	0.3	0.3

For the multiscale model, the displacement fields along the fiber's length direction y are expanded by Fourier series since they vary very fast in this direction. To set a specific wavelength of the microscopic fiber, the microscopic structure is represented by RVE₁₄ with half-wave number $q_0 = 2$. A small

transversal perturbation force $f = 1$ N is applied on the middle of the fiber in order to trigger the fiber buckling. RVE is meshed with 2 elements along the vertical direction and 3 elements along the horizontal direction based on 8-node quadrilateral elements and Gauss-Legendre quadrature with 9 integration points per element. The macroscopic structure is meshed with 10 elements along its length and width respectively, based on the same quadrilateral elements as RVE. This multiscale model is solved by ANM with the accuracy parameter $\epsilon = 10^{-6}$ and the truncation order of power series $N = 15$ [13].

Fig. 3 gives the compressive stress, along the fiber's length direction, distribution over the middle area of the plate when fiber buckling occurs. Figs. 3a and 3b are simulated by the fully meshed model and the multiscale model respectively, which shows good agreement. Besides, the Fourier slowly varying coefficient based microscopic model meshed by only a few elements has higher computational efficiency in comparison with a fine-meshed RVE, especially in the case of fibers with a large half-wave number. For example, in the case of $m = 1$, $n = 16$, $q_0 = 8$, our proposed microscopic model uses about total 150 *degrees of freedoms* (DOFs), while the fine-meshed RVE model needs about 4000 DOFs. Therefore, the computation cost could be reduced more than an order of magnitude. Considering the microscopic fiber buckling mode with more waves, the proposed model does not need more DOFs, while the fine-meshed RVEs will require much more DOFs. Moreover, the Fourier-related microscopic model can introduce a defect for a given wavelength without initial geometrical imperfections but a small perturbation force, which could reduce the work on geometrical modeling.

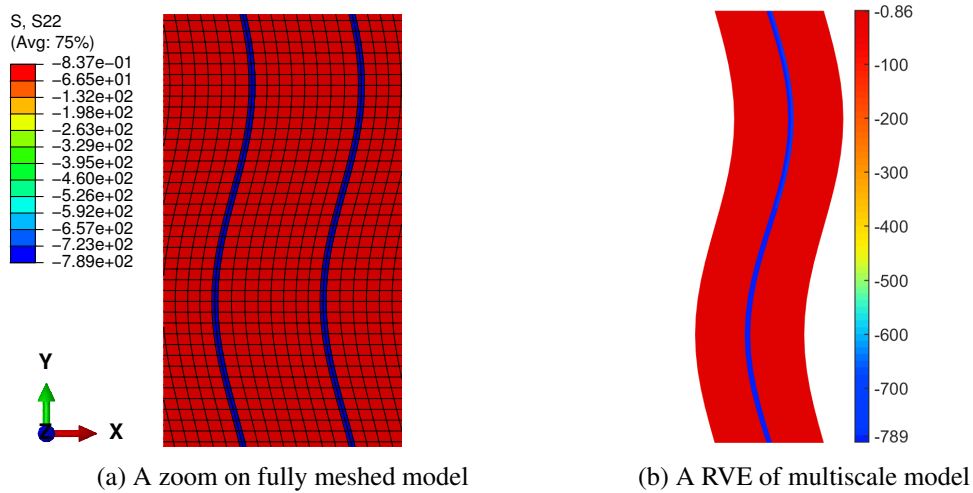


Figure 3: The compressive stress (MPa), along the fiber's length direction, distribution in the middle area of the composite simulated by the fully meshed model and the multiscale model respectively.

4 Conclusion

This paper presents a multiscale approach with an efficient microscopic model for simulating fiber reinforced composites. The microscopic structures are simulated by the proposed reduced model where all unknowns are expanded with Fourier slowly varying coefficients. This microscopic model can control the fibers' instability modes precisely and predict the compressive failure of composites accurately. Moreover, ANM is adopted as the solver to efficiently and accurately predict the nonlinear path. Both the accuracy and efficiency of the proposed multiscale approach are validated by a fully meshed ABAQUS model.

Acknowledgments

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References

- [1] N.A. Fleck, J.Y. Shu. *Microbuckle initiation in fibre composites: a finite element study*, Journal of the Mechanics and Physics of Solids, Elsevier, 1887–1918, 1995.
- [2] S. Drapier, J.C. Grandidier, M. Potier-Ferry. *A structural approach of plastic microbuckling in long fibre composites: comparison with theoretical and experimental results*, International Journal of Solids and Structures, Elsevier, 3877–3904, 2001.
- [3] K. Poulos, C.F. Niordson. *Homogenization of long fiber reinforced composites including fiber bending effects*, Journal of the Mechanics and Physics of Solids, Elsevier, 433–452, 2016.
- [4] S. Nezamabadi, M. Potier-Ferry, H. Zahrouni, J. Yvonnet. *Compressive failure of composites: A computational homogenization approach*, Composite Structures, Elsevier, 60–68, 2015.
- [5] F. Feyel. *A multilevel finite element method (FE^2) to describe the response of highly non-linear structures using generalized continua*, Computer Methods in Applied Mechanics and Engineering, Elsevier, 3233–3244, 2003.
- [6] V.G. Kousnetzova, M.G.D. Geers, W.A.M. Brekelmans. *Multi-scale second-order computational homogenization of multi-phase materials: a nested finite element solution strategy*, Computer Methods in Applied Mechanics and Engineering, Elsevier, 5525–5550, 2004.
- [7] S. Nezamabadi, J. Yvonnet, H. Zahrouni, M. Potier-Ferry. *A multilevel computational strategy for handling microscopic and macroscopic instabilities*, Computer Methods in Applied Mechanics and Engineering, Elsevier, 2099–2110, 2009.
- [8] R. Xu, C. Bouby, H. Zahrouni, T. Ben Zineb, H. Hu, M. Potier-Ferry. *3D modeling of shape memory alloy fiber reinforced composites by multiscale finite element method*, Composite Structures, Elsevier, 408–419, 2018.
- [9] R. Xu, Y. Hui, H. Hu, Q. Huang, H. Zahrouni, T. Ben Zineb, M. Potier-Ferry. *A Fourier-related FE^2 multiscale model for instability phenomena of long fiber reinforced materials*, Composite Structures, Elsevier, 530–539, 2019.
- [10] N. Damil, M. Potier-Ferry. *A generalized continuum approach to predict local buckling patterns of thin structures*, European Journal of Computational Mechanics, Taylor & Francis, 945–956, 2008.
- [11] Y. Liu, K. Yu, H. Hu, S. Belouettar, M. Potier-Ferry, N. Damil. *A new Fourier-related double scale analysis for instability phenomena in sandwich structures*, International Journal of Solids and Structures, Elsevier, 3077–3088, 2012.
- [12] Q. Huang, R. Xu, Y. Liu, H. Hu, G. Giunta, S. Belouettar, M. Potier-Ferry. *A two-dimensional Fourier-series finite element for wrinkling analysis of thin films on compliant substrates*, Thin-Walled Structures, Elsevier, 144–153, 2017.
- [13] B. Cochelin. *A path-following technique via an asymptotic-numerical method*, Computers & Structures, Elsevier, 1181–1192, 1994.
- [14] H. Abichou, H. Zahrouni, M. Potier-Ferry. *Asymptotic numerical method for problems coupling several nonlinearities*, Computer Methods in Applied Mechanics and Engineering, Elsevier, 5795–5810, 2002.