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Strongly coupled finite element framework for a thin fluid flow in contact interfaces

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Résumé — We developed a monolithic finite-element framework, which includes robust contact resolution algorithms, fluid-flow elements for solving Reynolds equation for the incompressible viscous flow and fluid-structure interface elements to apply fluid pressure on the solid. Additionally, we take into account the possibility of fluid entrapment in the contact interface and its pressurization using non-linearly compressible constitutive laws and formulate a novel trapped-fluid element.
Mots clés — fluid-structure interaction, contact, fluid entrapment.

1 Introduction

The problem of thin fluid flow in narrow interfaces between contacting or slightly separated deformable solids appears in different contexts : from tribological, engineering and biological applications to geophysical sciences. Rigorous handling of such problems requires resolution of a strongly nonlinear contact problem, which is further complicated by a multi-field coupling of essentially interrelated fluid and solid mechanics. Moreover, the free volume [1] between contacting surfaces depends on their initial geometry, which can be rather complex, may have deterministic features or, at a certain magnification, may be considered as randomly rough (self-affine) down to atomistic scale. Numerous applications of the problem of thin fluid flow in contact interfaces include sealing engineering, lubrication in elasto-hydrodynamic and mixed regimes, functioning of human joints. Such an interaction between fluids and solids in contact is also relevant for hydraulic fracturing, extraction of shale gas and oil from rocks and at larger scales in landslides, slip in pressurized faults and bazal sliding of glaciers

The problem of the thin fluid flow in contact interfaces belongs to a vast domain of fluid-structure interaction (FSI) problems, which involve deformation and/or motion of the solids interacting with the internal and/or external fluid. These problems are of very wide range, spanning from deformation of airplane wings and rotor blades subjected to the sub- or supersonic air flow [1, 2] to modelling of the blood flow [3, 4] and heart valves [5, 6], scaling up to suspended bridge instabilities under wind load [7], ship stability [8] or huge iceberg's capsize in water [9]. All these problems correspond to different space and time scales, operating conditions and other requirements, therefore a unified FSI approach fit for all cases does not exist, and rather different techniques have been developed for particular problems.

Many problems of the fluid-structure interaction, such as aeroelasticity and hemodynamics, correspond to the case of the high-Reynolds-number flow. Therefore, different mesh density, and often different time stepping, are required for the solid and fluid domains. Furthermore, the fluid domain evolves due to motion and deformation of solids. A number of methods have been used to overcome the associated computational complexity, such as arbitrary Lagrangian-Eulerian method [10, 11], fictitious domain method [12, 6], immersed boundary method [13, 14] and extended finite element method [15, 16]. On the contrary, fluid flow in contacting or slightly separated interfaces is usually of low Reynolds number and, moreover, the thickness of the fluid film is usually much smaller than other dimensions of the solid. In this case general Navier-Stokes equations could be readily simplified down to the Reynolds equation for the viscous flow [17]. This simplification permits to use compatible meshes for the fluid and the solid domains and, under assumption of constant pressure through the film thickness, define the Reynolds equation on the so-called lubrication surface, so that specific methods discussed above are not required,

1. By free volume, here, we mean the void separating the contacting surfaces.

see [18, 19]. This approach is also used in the current study.

2 Methodology

We developed a monolithic finite-element framework, which includes robust contact resolution algorithms, fluid-flow elements for solving Reynolds equation for the incompressible viscous flow and fluid-structure interface elements to apply fluid pressure on the solid. Additionally, we take into account the possibility of fluid entrapment in the contact interface and its pressurization using non-linearly compressible constitutive laws and formulate a novel trapped-fluid element. The computational framework includes image analysis algorithms to distinguish between contact, fluid flow and trapped fluid zones and is suitable for both one- and two-way coupling approaches.

We consider a problem of the thin fluid flow in contact interface between a solid body with an arbitrary surface geometry, given, for concreteness, by a function $z(x,y)$, and a rigid flat given by a plane $z=0$. Let us denote by Ω_s the deformable solid and by $\Gamma \subset \partial\Omega_s$ the part of its surface which represents the *potential* contact zone, i.e. defines the extent of the contact interface. This surface is subdivided into following parts according to the local status of the interface :

$$\Gamma = \Gamma^{c} \cup \Gamma^{fsi} \bigcup_{i=1}^{n_{tf}} \Gamma_i^{tf},$$

where Γ^{c} is the *active* contact zone where normal contact tractions are non-zero, Γ^{fsi} is part of the solid's surface which interacts with the flowing fluid and where the surface tractions are equal to the corresponding tractions in the fluid, $\Gamma_i^{tf}, i = 1 \dots n_{tf}$ are trapped fluid zones, i.e. parts of the surface Γ which are out of contact, but completely delimited by non-simply connected contact patches. Note that by definition $\Gamma^{fsi} \cap \Gamma_i^{t} = \emptyset \; \forall i = 1 \dots n_{tf}$, and $\Gamma_i^{t} \cap \Gamma_j^{t} = \emptyset \; \forall i, j = 1 \dots n_{tf}$ and $i \neq j$. Furthermore, we term by Γ^{f} the projection of Γ^{fsi} on the rigid plane $z=0$, which would serve as the lubrication surface where the Reynolds equation for the fluid flow will be defined. Note that we assume here that in the initial configuration $\Gamma^{c} = \emptyset$ and $n_t = 0$, so that the fluid flow zone occupies the whole interface. This assumption makes impossible appearance of non-contact zones, which do not belong to neither Γ^{fsi} nor to one of the trapped fluid zones in any deformed configuration. However, consideration of such a case (e.g. air bubbles entrapment) could be easily included into the framework.

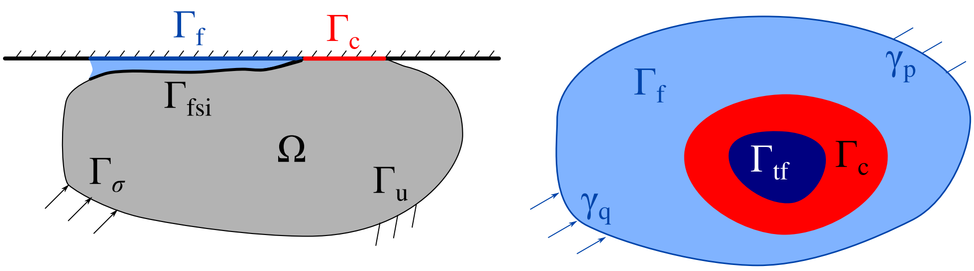

FIGURE 1 – Sketch of the problem under study : (a) side-view of the contact between a solid and a rigid plane with a fluid present in contact interface ; (b) 2D view of the contact interface with .

The deformation of the solid is governed by the balance of momentum equation complemented by the contact and boundary conditions, where the fluid pressure appears :

$$\begin{cases} \nabla \cdot \boldsymbol{\sigma}(\boldsymbol{u}) + \boldsymbol{f}_v = 0 \text{ in } \Omega_s \\ g_n(\boldsymbol{u}) \geq 0, \; \sigma_n(\boldsymbol{u}) \leq 0, \; g_n(\boldsymbol{u})\,\sigma_n(\boldsymbol{u}) = 0 \text{ on } \Gamma \\ \boldsymbol{u} = \boldsymbol{u}_0 \text{ on } \Gamma^{u} \\ \boldsymbol{n} \cdot \boldsymbol{\sigma} = \boldsymbol{\sigma}_0 \text{ on } \Gamma^{\sigma} \\ \boldsymbol{n} \cdot \boldsymbol{\sigma}(\boldsymbol{u}) = -\boldsymbol{t}_f \quad \text{at } \Gamma_{f} \cup \Gamma^{tf} \end{cases} \tag{1}$$

where the first line is the static local balance of momentum equation, the second line summarizes the Hertz-Signorini-Moreau conditions of the non-adhesive frictionless unilateral contact, next two lines are the classical Dirichlet and Neumann boundary conditions with a prescribed displacement $\boldsymbol{u}_0$ and surface traction $\boldsymbol{\sigma}_0$, and the last line corresponds to the traction $\boldsymbol{t}_f$ due to the fluid flow in the contact interface or due to the entrapped fluid (for the latter, only the normal component should appear).

The thin fluid flow in governed by :

$$\begin{cases} \nabla\cdot\left[g_n(\boldsymbol{u})^3\nabla p\right]=0 \quad \text{in } \Gamma^{\mathrm{f}} \\ p=p_0 \text{ on } \gamma^p \\ \boldsymbol{q}\cdot\boldsymbol{m}=q_0 \text{ on } \gamma^q \end{cases} \tag{2}$$

where the first line is the Reynolds equation for isoviscous incompressible Newtonian fluid, note that the tangential relative motion of the solid walls is not considered here whereas the normal approaching is assumed to be quasistatic, p is the fluid pressure field defined on the lubrication surface Γ^{f}, which is a projection of the fluid-structure interface Γ^{fsi} on the plane $z=0$, associated with the rigid flat ; the thickness of the channel is given be the normal gap g_n, second line is the Dirichlet boundary condition with a prescribed fluid pressure p_0, and the third line is the Neumann boundary condition with a prescribed fluid flux q_0, defined at $\gamma^p \subset \partial\Gamma^{\mathrm{f}}$ and $\gamma^q \subset \partial\Gamma^{\mathrm{f}}$, respectively ($\boldsymbol{m}$ is the outer normal to Γ^{f}).

The hydrostatic pressure p^t_i, developed in the i-th trapped fluid zone, is applied to the surface of the solid body as the normal traction :

$$\sigma_n=-p^t_i \quad \text{on } \Gamma^t_i, \quad i=1\dots n_t. \tag{3}$$

If the fluid in trapped zones is assumed incompressible, then the following constraint on the fluid volume V_i in the i trapped zone is considered :

$$V_i(\boldsymbol{u})=V_{i0}, \tag{4}$$

where V_{i0} is the volume of the fluid in the i-th trapped pool at the exact moment when it was formed. Furthermore, if the trapped fluid compressibility is taken into account, then instead of (4) a constitutive relation between the fluid pressure and its volume change is to be provided. In the linear case with constant bulk modulus K of the fluid it is given by :

$$p^t_i=p^t_{i0}+K\left(1-\frac{V_i(\boldsymbol{u})}{V_{i0}}\right), \tag{5}$$

where p^t_{i0} is the initial pressure of the trapped fluid corresponding to the volume V_{i0}. In the case of pressure-dependent fluid bulk modulus $K=K_0+K_1\,p^t_i$, the relation becomes nonlinear and takes the following form :

$$p^t_i=\left(\frac{K_0}{K_1}+p^t_{i0}\right)\left(\frac{V_i(\boldsymbol{u})}{V_{i0}}\right)^{-K_1}-\frac{K_0}{K_1}. \tag{6}$$

In this work, equations (1),(2) and (4) or (5) or (6) are solved within a monolithic computational framework including both fluid and solid degrees of freedom. The mortar method for contact coupled with monolithic augmented Lagrangian method is used. In case of incompressible trapped fluid model, we use the method of Lagrange multipliers, for the case of compressible trapped fluid, a non-linear penalty method is adapted. The computational difficulty of the coupled implementation consists in accurate determination of the status of interfacial elements (contact (stick / slip) / flowing fluid / trapped fluid) : this difficulty raises from the fact that trapped zones can appear, split, disappear or join together ; being combined with periodic boundary conditions, which are natural for many relevant applications, this permanent status switch is algorithmically not easy to track. Moreover, it affects the convergence of the iterative Newton method, which can be ensured to be quadratic only when all the statuses remain fixed. Nevertheless, the convergence is achieved in all considered problems even those which include numerous zones of trapped fluid.

3 Results and discussions

First, in two-dimensional set-up, a behaviour of a pressurized fluid trapped between deformable solid with a wavy surface and a rigid flat was studied [20]. We showed how the evolution of the real contact area and of the global coefficient of friction under increasing external load depend on material properties of the fluid, as well as on that of the solid, and on the slope of the surface profile. The opening of the trap by the fluid in presence of friction was also analyzed. From computational point of view this study is interesting by a possibility to assume an arbitrary overlap of fluid and the contact interfaces.

In the full three-dimensinal set-up, the proposed framework was also used to study the fluid flow in the interface between a rigid flat and a deformable solid with a model geometry or random self-affine rough surface. We derived an approximate analytical solution for the fluid flow across a wavy contact interface, which was compared with our numerical results [21]. Finally, we showed that for a range of physically relevant parameters, one-way coupling underestimates the interface permeability and the critical external load needed to completely seal the interface, compared to the two-way approach. A refined phenomenological law for macroscopic permeability of rough contact interfaces was proposed.

In Fig. 2 we demonstrate results obtained for a strongly coupled fluid flow in the contact interface between a rigid flat and a solid with a representative surface roughness.

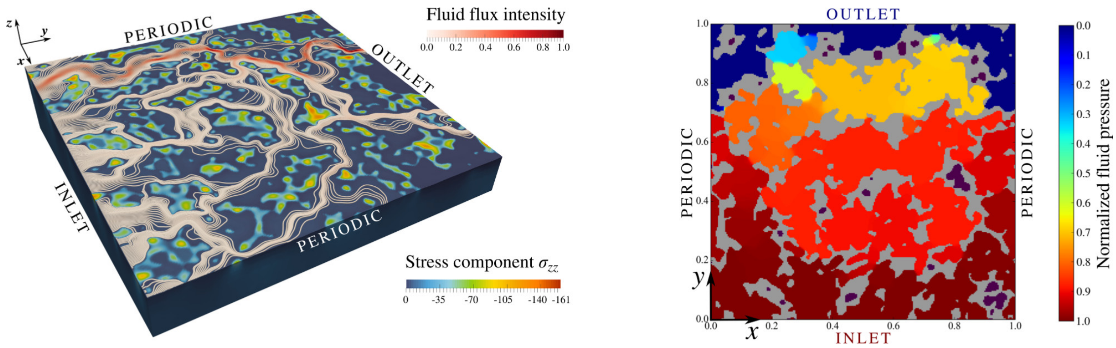

FIGURE 2 – Fluid flow through contact interface between a solid with self-affine rough surface and a rigid flat ($z = 0$, not shown). Left : bulk view with color representing σ_{zz} component of the stress tensor and streamlines with color showing the fluid flux intensity (norm of the flux vector, divided by the max. observed value). Right : interface view with color representing the fluid pressure (normalized by the inlet pressure), contact patches are shown in gray and all trapped fluid zones are purple.

4 Conclusion

The originality of the developed monolithic computational framework to treat problems of interfacial fluid flow in contact interfaces is that it can handle at the same time three possible interface phenomena : namely, (1) frictional contact, (2) fluid flow and (3) effect of fluid entrapment and its pressurization using relevant physical models. All algorithmical details are provided and few interesting physical model-problems are solved. The prospective work focuses on the inclusion of tangential motion of contacting solids, which would require (1) modification of the used Reynolds equation, (2) adding cavitation effect and (3) adding a criterium of transition to boundary lubrication, which would indicate intimate mechanical contact between surfaces.

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