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Extended mortar methods to treat tying and contact interface problems

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Résumé — We present a unified framework, called MorteX to treat embedded interface problems in the context of tying and contact. This framework is a combination of the X-FEM and mortar methods, whereby Lagrange multipliers are used to enforce interface constraints. As known, mixed formulations are prone to mesh locking which is characterized by the emergence of spurious oscillations in the vicinity of the tying interface. To overcome this inherent shortcoming, we suggest a new coarse-grained interpolation of Lagrange multipliers. This technique consists in selective assignment of Lagrange multipliers on nodes of the mortar side. The optimal choice of the coarse-graining spacing is guided solely by the mesh-density contrast between the mesh of the mortar side and the number of blending elements of the host mesh. The applicability of the proposed stabilization techniques is demonstrated for both the tying and contact problems solved within the MorteX framework.
Mots clés — embedded interface, MorteX method, stabilization, mortar method, X-FEM, spurious oscillations, over-constraining.

1 Introduction

The finite element method (FEM) is used to solve a wide range of physical and engineering problems. Based on a variational formulation and a discretized representation of the geometry, this method is extremely flexible in handling complex geometries, non-linear and heterogeneous constitutive equations and multi-physical/multi-field problems. A classification of finite element models can be proposed based on the strategy to represent the boundary of the computational mesh. Classical FE meshes fall into the category of "boundary fitted" (BF) methods, where the boundaries of the physical and computational domains coincide [Fig. 1(a)]. Alternatively, for "embedded/immersed boundary" (EB) methods, the computational domain is a mesh or a Cartesian grid hosting another physical domain [Fig. 1(b)].

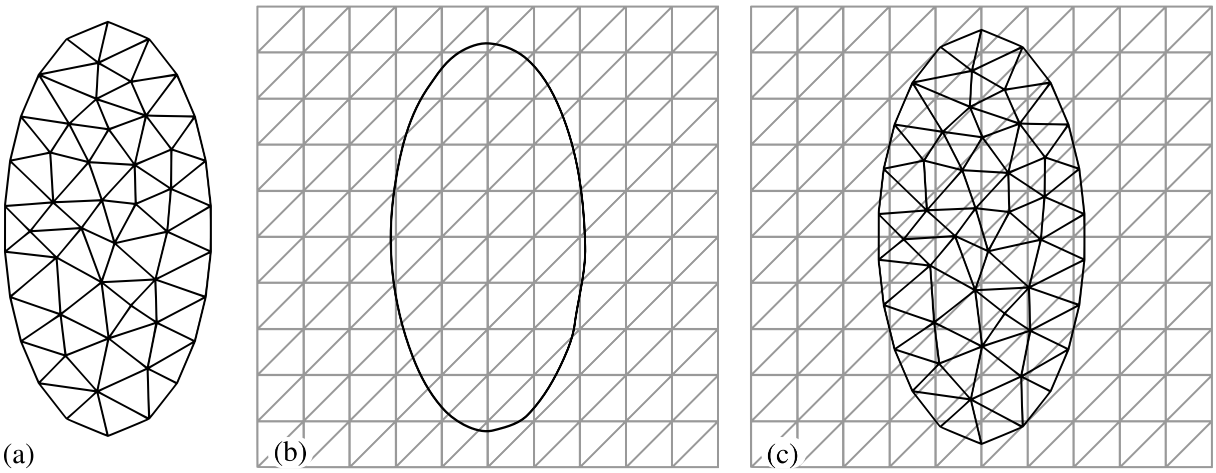


FIGURE 1 – Illustration of meshes with (a) fitted boundary ; (b) embedded boundary (for example, level set) ; (c) embedded mesh.

Here, in the MorteX framework we consider a particular combination of BF and EB methods shown in Fig. 1(c) which deals simultaneously with two or several superposed meshes. This is achieved by using the features of mortar methods, and of X-FEM in the context of void/inclusion modeling.Within, this framework we propose to solve the two classes of interface problems, namely : tying and contact problems.

The proposed methods are developed and implemented in the in-house finite element suite Z-set. Numerous numerical examples are considered to validate the implementation and demonstrate its robustness, performance and accuracy. In addition, the MorteX framework is used to treat frictional wear problems. Within this framework the contact surface evolution resulting from material removal due to wear is modeled as an evolving virtual surface. The MorteX method circumvents the need for complex remeshing techniques to account for contact surface evolution.

2 Methodology

2.1 Extended finite element method

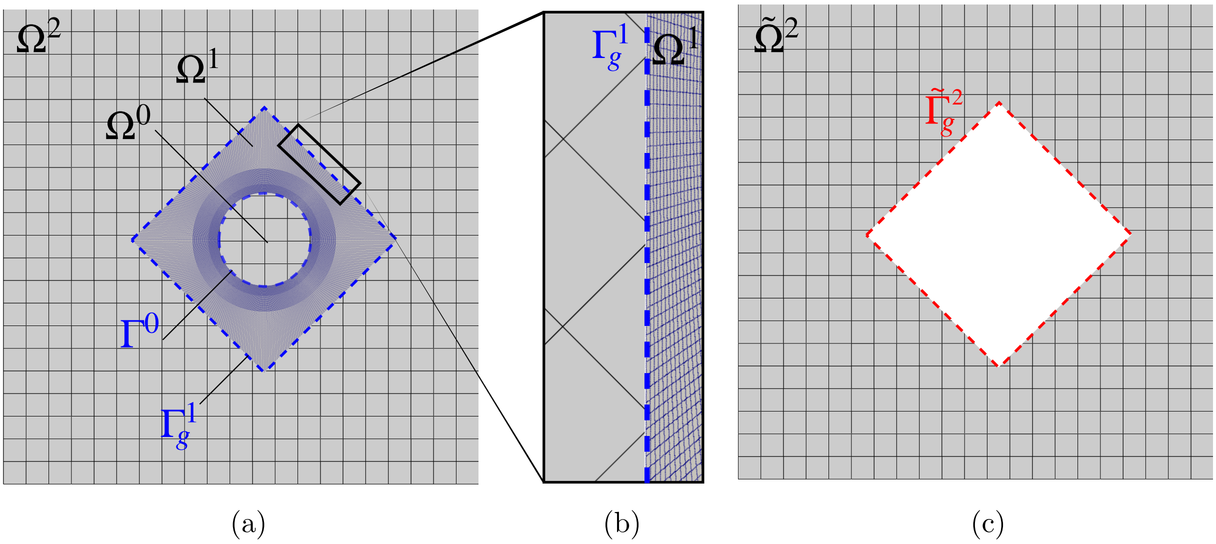


FIGURE 2 – (a) Two overlapping meshes : the host Ω^2 and the patch Ω^1 with a circular hole Ω^0 are tied along interface Γ_g^1 ; (b) zoom on the interface between the host and patch meshes ; (c) effective volume of the host mesh $\tilde{\Omega}^2 = \Omega^2 \setminus \{\bar{\Omega}^0 \cup \bar{\Omega}^1\}$.

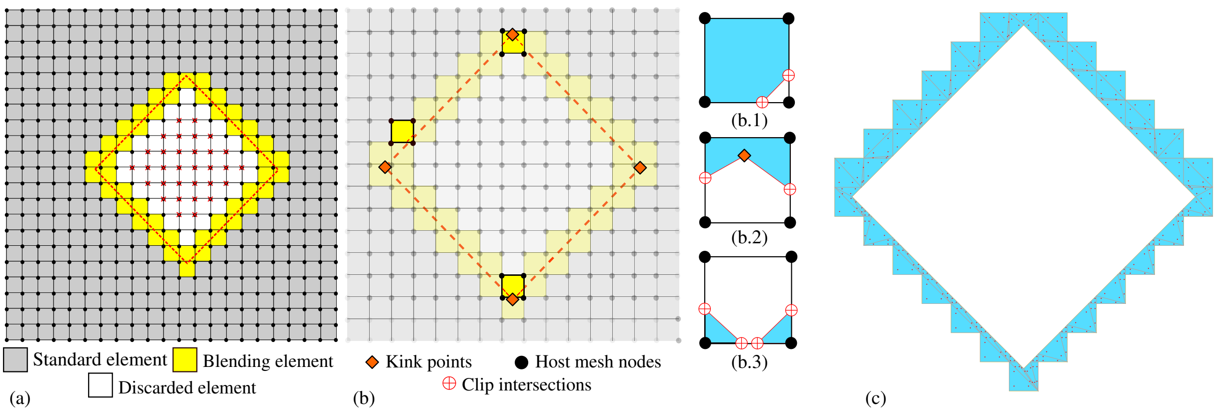


FIGURE 3 – (a) Element classification in X-FEM framework ; (b) clipping of blending elements by $\tilde{\Gamma}_g^2$, the volume colored in blue in (b.1-3) is the effective volume of integration ($\tilde{\Omega}^e$) : (b.1) a convex polygon, (b.2) a non-convex polygon, (b.3) disjoint polygons ; (c) selective integration is carried out over re-triangulated blending elements with reinitialized Gauss integration points (shown in red).

The virtual surface $\tilde{\Gamma}_g^2$ of the host domain is treated as an internal discontinuity. This is modeled within the X-FEM framework, thereby nullifying the presence of the overlap region $\left(\bar{\Omega}^1 \cup \bar{\Omega}^0\right) \cap \Omega^2$ in the domain Ω^2 [see Fig. 2(c)]. The X-FEM relies on enhancement of the FEM shape functions used to interpolate the displacement fields. Here the enrichment functions describing the field behavior are incorporated locally into the finite element approximation. This feature allows the resulting displacement to capture discontinuities. The subdivision of the host mesh is defined by indicator function $\phi(\vec{X}) : \mathbb{R}^{\text{dim}} \to \{0,1\}$ (where $\vec{X}$ is the spatial position vector in the reference configuration in domain Ω^2) [1]. The indicator function is non-zero only in the non-overlapping part of domain Ω^2 :

$$\phi(\vec{X}) = \begin{cases} 1, & \text{if } \vec{X} \in \tilde{\Omega}^2; \\ 0, & \text{elsewhere.} \end{cases}$$

The discontinuity surface $\tilde{\Gamma}_g^2$ can be seen as a level-set defined as follows :

$$\tilde{\Gamma}_g^2 = \left\{ \vec{X} \in \Omega^2 : \nabla\phi(\vec{X}) \neq 0 \right\}$$

As a result, the indicator function $\phi(\vec{X})$ partitions the elements of the host domain Ω^2 into three distinct categories [Fig. 3(a)], namely standard elements, blending elements and discarded elements.

In practice, the enrichment of shape functions in case of void/inclusion problem can be simply replaced by a selective integration scheme [2]. For the standard elements, there is no change in volume of integration and the discarded elements are simply excluded from the volume integration procedure. In order to obtain the effective volume of integration for each blending element, we perform the clipping of the blending elements by the discretized surface $\tilde{\Gamma}_g^2$ [Fig. 3(b)] . The clipping of a single element could result in one or several various polygons both convex, and non-convex, which represent the effective volumes of integration.

To selectively integrate the internal virtual work in the effective volume only, the resulting polygons are virtually remeshed into standard convex elements (for example, triangles). Note that this remeshing is merely performed to use a Gauss quadrature for integration [Fig. 3(c)], and does not imply the creation of additional degrees of freedom, and as such does not change the topological connectivity of nodes. The displacement field is evaluated using the standard shape functions and the original DoFs ; only the integration is changed. To carry out this remeshing, we applied the ear clipping triangulation algorithm to the polygons. The DoFs associated with the elements outside the integration domain $\tilde{\Omega}^2$ [marked with red crosses in Fig. 3(a)] are removed from the global system of equations.

2.2 Mortar method

The mortar finite element discretization will be used to impose the tying and contact constraints in a weak sense. Within the mortar discretization framework, the tied domains are classified into mortar and non-mortar sides. The superscript "1" refers to the mortar side of the interface and "2" to the non-mortar side ; the former stores the Lagrange multipliers (dual DoFs) in addition to displacement degrees of freedom (primal DoFs) [3]. In comparison to the classical schemes, in order to impose constraints along the real-embedded interface, few adaptations are to be made for the evaluation of mortar integrals [4, 5]. Fig. 4(a), show the integration scheme for the tying problem in MorteX framework. In contrast to the classical schemes [3], here projections are not required to determine the limits of mortar domain S^{el}. The clip intersections/kink points (the mortar nodes that lie inside a host element) are the limits of S^{el}, and are determined by the clipping process. However the integration scheme for the contact problem remains largely similar to the classical mortar scheme, with an exception that in MorteX framework the clip intersections/kink points are projected instead of non-mortar side nodes [see Fig. 4(b)].

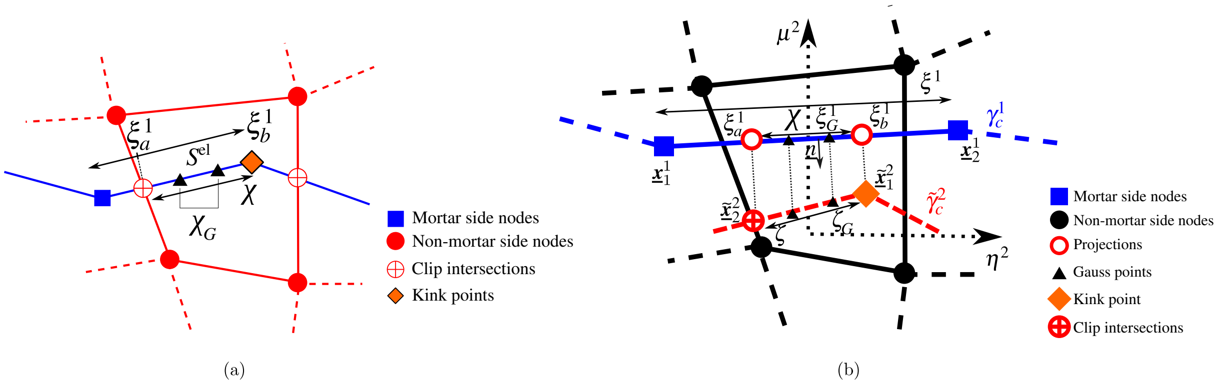


FIGURE 4 – Example integration domain in MorteX framework : (a) tying ; (b) contact.

3 Numerical issues

The standard Lagrange multiplier methods is used to impose the equality constraint of the mesh tying problem, while the augmented Lagrangian method is invoked to resolve the inequality contact constraints. These Lagrange multiplier based methods represent the so-called mixed formulations [6]. The choice of Lagrange multipliers functional space strongly affects the convergence rate and can lead to loss of accuracy in the interfacial tractions [see Fig. 6, 7]. These difficulties arise from a locking type phenomena reported for mixed variational formulations using the standard Lagrange interpolations (SLI) as a result of non-satisfaction of Ladyzhenskaya-Babuška-Brezzi (LBB) [8, 6]. In particular, the issues resulting from the imposition of Dirichlet boundary conditions using Lagrange multipliers methods has been a topic of interest in various domains, such as the classical FEM [7], Interface-enriched Generalized Finite Element Method (IGFEM) [9], the fictitious domain methods [10], the mesh free methods [11], etc. In addition this problem has been dealt extensively within the context of the X-FEM. In [12, 13, 14], the authors propose a strategy to construct an optimal Lagrange-multiplier space for the embedded interfaces which permits to apply Dirichlet boundary conditions. As opposed to the strategy of modifying the Lagrange multiplier spaces, the authors in [15] propose a stabilization method to mitigate the oscillatory behaviour of the standard spaces.

In this work, we introduce the coarse grained interpolations (CGI) for the Lagrange multipliers and triangulation of blending elements as stabilization strategies to mitigate the emergence of spurious oscillations within the MorteX framework for mesh tying problems. The coarse grained interpolations is an extension of the strategy of modifying Lagrange multiplier spaces [12, 13, 14], which allows us to address specific problems of mesh-locking, which are inherent to mortar methods for overlapping domains, particularly in presence of a strong contrast of material properties and mesh densities in the vicinity of the interface. This modified Lagrange interpolations results in relaxing the over-constraining. In addition, the manifestation of mesh locking effects when imposing contact constraints using Lagrange multipliers will be demonstrated. Such mesh-locking effects were never reported to the best of our knowledge. Also the applicability of the stabilization strategies to the contact problems both in the context of classical mortar and MorteX methods will be demonstrated.

3.1 Coarse grained interpolations

Coarse graining of Lagrange multiplier interpolation functions enables to reduce the number of constraints and thus improves the problem stability. In this approach, not every mortar node is equipped with a Lagrange multiplier. Therefore, the interpolation functions become non-local, i.e. they span more than one mortar segment. For this purpose, we choose a 1D parametric space $\xi^{CG} \in [-1,1]$, spanning multiple mortar-side segments. Such parametrization can be chosen such that length L^i of the corresponding super-segment in the physical space is comparable to the size of host elements. As shown in Fig. 5, the mortar-surface is segmented into three super-segments of lengths L^1, L^2 and L^3. The end nodes of these segments are termed the "master" nodes (they carry the dual DoFs $\vec{\lambda}$), other mortar-nodes are termed "slave" nodes. We introduce the local coarse-graining parameter κ that determines the number of segments contained in a super-segment, and thus $(\kappa - 1)$ determines the number of slave nodes per super-segment.

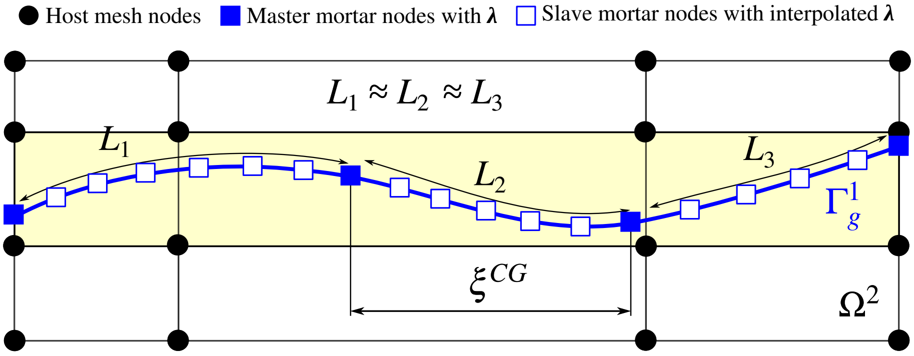


FIGURE 5 – An example illustrating the embedded surface Γ_g^1 cutting through the blending elements (shaded in yellow) of the coarser host mesh. The coarse graining of Lagrange multipliers can be implemented with respect to the local.

The positive effect of curbing the spurious oscillations and enabling to better capture the reference solutions, in case of tying and contact are shown in the Fig. 6, 7, respectively.

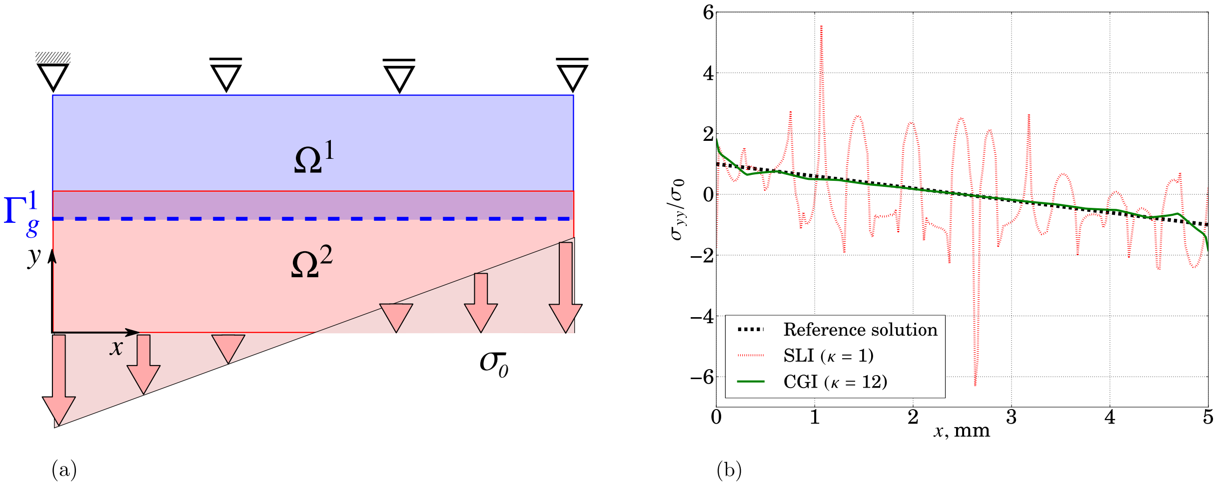
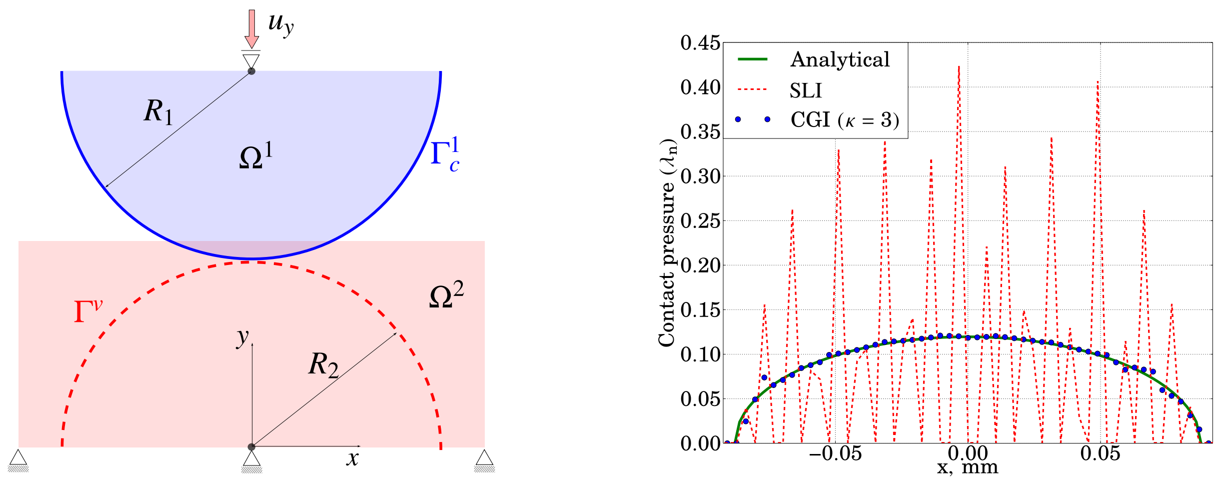


FIGURE 6 – Illustration of mesh locking in bending test : (a) problem setting ; (b) plots of interfacial stresses σ_{yy} along the tying surface Γ_g^1.



(a) (b)

FIGURE 7 – Illustration of mesh locking in Hertz contact test : (a) problem setting ; (b) plots of interfacial stresses σ_{yy} along the contact surface Γ_c^1.

4 Conclusion

In this work we present the two dimensional MorteX framework. This framework enables handling such interface problems as mesh tying between overlapping domains as well as frictional contact between embedded (virtual) boundaries. The inherent stability issues resulting from the Lagrange multiplier based mixed formulations, and their manifestations in the form of spurious oscillations are illustrated under various problem settings. Two stabilization techniques, namely automatic triangulation of blending elements and coarse-grained Lagrange multiplier interpolation are proposed to overcome these adverse effects. The novel contribution of coarse graining Lagrange multipliers enables to efficiently avoid over-constraining in the interface and obtain smooth and oscillation-free stress fields for arbitrary mesh and material contrast.

The results obtained in terms of having a stable, accurate and flexible formulation are encouraging both for tying and contact. A natural course would be to extend this framework to three dimensions. This however, poses challenges on the technical front. For example, finding an optimal coarse graining parameter κ in case of 3D is not as trivial as in 2D but is still feasible. In addition, the complexity of clipping (non-convex polygons) algorithms for selective integration increases.

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