CSMA 2019
14ème Colloque National en Calcul des Structures
13-17 Mai 2019, Presqu'île de Giens (Var)

Non incremental LATIN-PGD solver for non-linear vibratoric dynamics problems.

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Résumé —
The LATIN-PGD method is a fast non-linear and non incremental solver generally applied in solid mechanics that use on their formulation a model reduction technique called *Proper Generalize Decomposition*. To date, the method has only been applied for solving the response of structures under quasistatic conditions, leaving aside the inertial effects. This abstract present an extension of the LATIN-PGD for vibratoric dynamics problems while keeping the advantages of the solver's strategy. The method is verified studying the bending response of a plate.
Mots clés — LATIN, PGD, Solid Mechanics, Dynamics.

1 Introduction

The increasing complexity of the numerical models used to predict the (often non-linear) seismic behaviour of structures impose large computation time for solving the partial differential equations of the reference problem.

To reduce the computational time, dedicated strategies, sometimes referred to as "model reduction techniques", are usually considered. Among the most effective, the LATIN-PGD method has proved particularly efficient for treating large type of problems for structures under quasi-static loading and with typical elastic-visco-plastic behaviour [4] or more recently [5] on reinforced concrete modelling. For evolutionary problems, the LATIN approach consists in a sequence of (i) *local steps* where constitutive relations are solved for each Gauss point of the finite element formulation, and (ii) *global steps* where the equilibrium on the whole space-time domain is imposed. The most numerically expensive task is by far the re-imposition of global equilibrium at step (ii) ; however, this step is linear and its solution can be approached using a low rank approximation, known as the *Proper Generalized Decomposition* (PGD), which greatly improves the numerical performance of the approach and makes it non-incremental in time by nature.

In spite of the great variety of applications where the method has proven its efficiency, all of them has been done under quasi-static conditions, a condition that does not correctly model certain physical phenomena such as the response of massive structures under seismic excitations, or the inertial influence when a natural frequency of a structure is excited. The present abstract gives an extension of the classic LATIN-PGD method for vibratoric dynamics problems in order to take into account the inertial effects over the non-linear response of the structure.

2 Reference problem and the LATIN-PGD method in dynamics

The reference problem is shown in Figure 1 and correspond to a dynamic isothermal evolution of a structure defined over the space-time domain $\Omega \times I$, with Ω the space domain and $I = [0, T]$ the total interval of time considered. The structure is subjected to prescribed

body forces f, to boundary forces f^N over $\partial_N\Omega$ and to prescribed displacements u^D over $\partial_D\Omega$. The volumetric density of the structure is given by ρ.

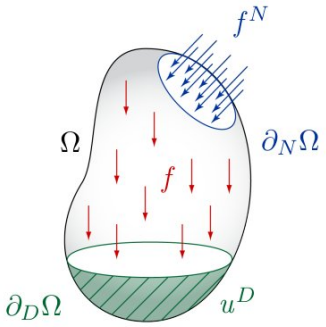


FIGURE 1 – Reference problem.

The solution is given by the set $\mathcal{S} = (\dot{\varepsilon}_p, \sigma)$, where $\dot{\varepsilon}_p$ denote the tensor of the rate of plastic deformation and σ the Cauchy stress tensor.

The resolution of the reference problem has three difficulties; first being satisfying the global equilibrium of the structure, the second being taking into account the non-linear elastic law, and finally to solve the non-linear evolution equations. The LATIN method is a non-incremental solver in time that tackles these sets of equations iteratively on the whole time-space domain. The algorithm is initialized by the elastic solution of the reference problem (solution $\mathcal{S}_0$), and then plasticity and (eventually) internal variables corrections are added iteratively. For that purpose two spaces are introduced based on a sub-division of the set of equations that must be solved, the first is the space $\boldsymbol{A_d}$ which belongs to the manifold of the admissibility conditions and the linear state laws. The second is a non-linear space $\boldsymbol{\Gamma}$ which belongs to the manifold of the evolution equations.

In order to find the solution, an iterative algorithm is used, which consists, at each iteration, of a non-linear local stage and a linear global stage, seek alternatively an approximation of the solution field "$\mathcal{S}$" over $\boldsymbol{\Gamma}$ and $\boldsymbol{A_d}$ as shown in Figure 2. The relation between the two spaces are governed by linear operators called search directions. The iterative algorithm can be represented as follows :

$$\mathcal{S}_0 \in \boldsymbol{A_d} \longrightarrow \hat{\mathcal{S}}_{1/2} \in \boldsymbol{\Gamma} ... \longrightarrow \hat{\mathcal{S}}_{n+1/2} \in \boldsymbol{\Gamma} \longrightarrow \mathcal{S}_{n+1} \in \boldsymbol{A_d} ... \longrightarrow \mathcal{S} \in \boldsymbol{A_d} \cap \boldsymbol{\Gamma}. \tag{1}$$

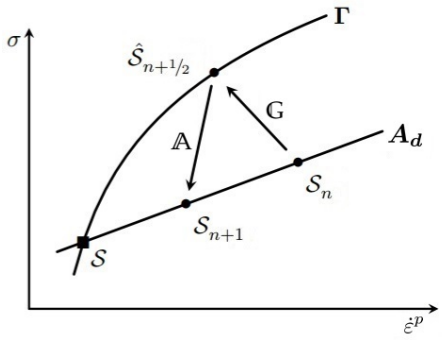


FIGURE 2 – Non-linear space $\boldsymbol{\Gamma}$ and linear space $\boldsymbol{A_d}$ representation of the LATIN method.

As seen from Figure 2, the final solution correspond to the intersection of both spaces. From herein we consider that the continuous problem is approximated using classical finite elements in space and time.

2.1 Constitutive relations

To fix the idea, in this abstract is considered the case of Chaboche's type perfect elastic-visco-plastic formulation [3], although more complex constitutive relations could be considered. The state equation is given by

$$\sigma = \mathbb{K} : \varepsilon_e \tag{2}$$

where $\mathbb{K}$ and ε_e are the Hooke tensor and the elastic deformation tensor respectively.
The evolution equation is given by :

$$\dot{\varepsilon}^p_{ij} = k\langle f^p \rangle^{\tilde{n}}_+ \left[\frac{3}{2} \frac{\sigma^D}{\sqrt{\frac{3}{2}\sigma^D : \sigma^D}} \right] \tag{3}$$

where $f^p = \sqrt{\frac{3}{2}\sigma^D : \sigma^D} - \sigma_y$, k, $\tilde{n}$ constants of the material, σ^D and σ_y the deviatoric stress tensor and the yield stress respectively.
The evolution equation (3) can be written in the more condensed form as :

$$\dot{\varepsilon}^p = \mathbb{B}(\sigma) \tag{4}$$

2.2 Initial Elastic solution $\mathcal{S}_0$

The procedure starts by calculating the dynamic elastic solution of the reference problem. This is easily done by using the modal base of the structure. The elastic solution must satisfy all the boundary conditions (Newman and Dirichlet) and is considered as the initial solution over the linear space $\boldsymbol{A_d}$.

2.3 Greedy process for the resolution of the non-linear reference problem

Lets consider that we have already calculated the linear solution at iteration n and we seek the solution at iteration $n+1$, both over the space $\boldsymbol{A_d}$, to do so, it must be follow the following stages.

2.3.1 Local non-linear stage over $\boldsymbol{\Gamma}$

Once the linear solution on the space $\boldsymbol{A_d}$ at iteration n is computed, we calculate the local non-linear solution on the space $\boldsymbol{\Gamma}$ by using the following ascent direction :

$$\left[\hat{\dot{\varepsilon}}^p_{n+1/2} - \dot{\varepsilon}^p_n \right] + \mathbb{G} : \left[\hat{\sigma}_{n+1/2} - \sigma_n \right] = 0 \tag{5}$$

where any positive operator can be use for $\mathbb{G}$. In this abstract and as in the literature in general it is chosen a constant ascent direction as follows, this means, we impose :

$$(\mathbb{G})^{-1} = 0 \tag{6}$$

this is traduced in $\hat{\sigma}_{n+1/2} = \sigma_n$ and therefore we calculate explicitly over all the Gauss points in $\Omega \times I$ the expression $\left[\hat{\dot{\varepsilon}}^p_{n+1/2} \right] = \mathbb{B}(\sigma_n)$.

The tangent operator $\mathbb{A}$ is also calculated at this step and is given as :

$$\mathbb{A} = \frac{\partial \mathbb{B}}{\partial \sigma} = k\tilde{n}\langle f^p \rangle^{\tilde{n}-1}_+ \left(\frac{\frac{3}{2}\sigma^D_n}{J_2} \otimes \frac{\frac{3}{2}\sigma^D_n}{J_2} \right) + k\langle f^p \rangle^{\tilde{n}}_+ \frac{\frac{3}{2}\left(J_2 \mathbb{I} - \frac{3}{2}\frac{\sigma^D_n \otimes \sigma^D_n}{J_2} \right)}{J_2^2} \tag{7}$$

with $J_2 = \sqrt{\frac{3}{2}\sigma^D_n : \sigma^D_n}$ and $\mathbb{I}$ the identity matrix. The tangent operator is calculated also explicitly over all the Gauss points in space and time.

2.3.2 Global linear stage over A_d

At the global stage, apart from the verification of the state laws and admissibility conditions, the solution set $\mathcal{S}_{n+1} \in \boldsymbol{A_d}$ must also satisfy the descent search direction :

$$\left[\dot{\varepsilon}^p_{n+1} - \hat{\dot{\varepsilon}}^p_{n+1/2}\right] - \mathbb{A} : \left[\sigma_{n+1} - \hat{\sigma}_{n+1/2}\right] = 0 \tag{8}$$

At this stage the quantities of interest are represented in a corrective form as a separable space and time functions at iteration $n+1$ as :

$$\begin{aligned} \dot{\varepsilon}^p_{n+1}(x,t) &= \Delta\dot{\varepsilon}^p_{n+1} + \dot{\varepsilon}^p_n = \mathbb{E}^p_{n+1}(x)\dot{\lambda}_{n+1}(t) + \sum_{i=1}^{n} \mathbb{E}^p_i(x)\dot{\lambda}_i(t) \\ \sigma_{n+1}(x,t) &= \Delta\sigma_{n+1} + \sigma_n = \mathbb{C}_{n+1}(x)\lambda_{n+1}(t) + \sum_{i=1}^{n} \mathbb{C}_i(x)\lambda_i(t) \end{aligned} \tag{9}$$

The calculation of the spatial and temporal function is performed iteratively as a fixed-point algorithm, calculating first the spatial functions assuming known the temporal function, and second, calculating the temporal function given the spatial functions.

Dynamic space problem :

For the present section lets introduce the following admissibility spaces :

The space $\mathcal{U}^S(\Omega, u^D)$ of kinematically admissible displacement such that :

$$u(x,t=0) = u_0, \;\; \dot{u}(x,t=0) = \dot{u}_0 \;\text{ and }\; u(x,t) = u^D \;\; \forall x \in \partial_D\Omega \tag{10}$$

Also the space $\mathcal{U}^T = L^2(I;\mathbb{R})$ such that :

$$L^2(I;\mathbb{R}) = \left\{ v : I \to \mathbb{R} | \int_I \left\| v(t) \right\|^2_S dt < \infty \right\} \tag{11}$$

The space-time solution search on $\mathcal{U}$ is identified with the tensorial product $\mathcal{U}^S \otimes \mathcal{U}^T$.

Lets also define the space $\mathcal{F}(f, f^N)$ of *dynamically admissible* stress tensors such that σ verify the following weak formulation of the equilibrium equation :

$\forall v \in \mathcal{U}^S(\Omega,0) \otimes \mathcal{U}^T(I)$,

$$\int_{I\times\Omega} \sigma : \varepsilon(\dot{v})\, d\Omega dt = -\int_{I\times\Omega} \rho\ddot{u}\cdot\dot{v}\, d\Omega dt + \int_{I\times\Omega} f\cdot\dot{v}\, d\Omega dt + \int_{I\times\partial_N\Omega} f^N\cdot\dot{v}\, dS dt \tag{12}$$

The space of *kinematically admissible* strain tensor in turn, denoted $\mathcal{E}(u_0, \dot{u}_0, u^D)$, contains strain tensor fields such that $\exists u \mid \varepsilon = 1/2(\nabla u + \nabla^T u)$ and that verifies initial and Dirichlet conditions, that write in a weak sense :

$\forall \bar{\sigma}^* \in \mathcal{F}(0), \;\; \forall v \in \mathcal{U}^S(\Omega,0) \otimes \mathcal{U}^T(I)$

$$-\int_{\Omega\times I} \bar{\sigma}^* : \varepsilon(u)\, d\Omega dt \;+\; \int_{\partial\Omega_D\times I} \bar{\sigma}^* n.u^D\, dS dt \;=\; \int_{\Omega\times I} u\cdot\rho\ddot{v}\, d\Omega dt \tag{13}$$

Now lets consider two solutions at the global stage $\mathcal{S}_n$ and $\mathcal{S}_{n+1}$ that verifies the admissibility conditions of equation (12), then lets subtract both equilibrium equations, that is to say, lets compute $\bar{\mathcal{S}}_{n+1} = \mathcal{S}_{n+1} - \mathcal{S}_n$. The last procedure gives :

$\forall v \in \mathcal{U}^S(\Omega,0) \otimes \mathcal{U}^T(I)$,

$$\int_{I\times\Omega} (\sigma_{n+1} - \sigma_n) : \varepsilon(\dot{v})\, d\Omega dt = -\int_{I\times\Omega} \rho(\ddot{u}_{n+1} - \ddot{u}_n)\cdot\dot{v}\, d\Omega dt \tag{14}$$

that can be written in function of the corrections $\Delta\sigma_{n+1}$ and Δu_{n+1} as :
$\forall v \in \mathcal{U}^S(\Omega,0) \otimes \mathcal{U}^T(I)$,

$$\int_{I\times\Omega} (\Delta\sigma_{n+1}) : \varepsilon(\dot{v})\, d\Omega dt = -\int_{I\times\Omega} \rho(\Delta\ddot{u}_{n+1}) \cdot \dot{v}\, d\Omega dt \tag{15}$$

Due to the kinematic admissibility to zero of $\Delta\dot{\varepsilon}_{n+1}$ we have :

$\forall v \in \mathcal{U}^S(\Omega,0) \otimes \mathcal{U}^T(I), \quad \forall \bar{\sigma}^* \in \mathcal{F}(0)$,

$$\int_{I\times\Omega} \Delta\dot{\varepsilon}_{n+1} : \bar{\sigma}^* d\Omega dt = -\int_{I\times\Omega} \rho\Delta\dot{u}_{n+1} \cdot \ddot{v}\, d\Omega dt \tag{16}$$

by using the strain partition relation :

$$\Delta\dot{\varepsilon}_{n+1} = \Delta\dot{\varepsilon}^p_{n+1} + \mathbb{K}^{-1} : \Delta\dot{\sigma}_{n+1} \tag{17}$$

and using the search direction (8) to find an expression of $\Delta\dot{\varepsilon}^p_{n+1}$, we obtain :

$$\Delta\dot{\varepsilon}^p_{n+1} = \mathbb{A} : \Delta\sigma_{n+1} - \bar{\Delta}_{n+1} \tag{18}$$

with $\bar{\Delta}_{n+1} = \mathbb{A} : \left(\hat{\sigma}_{n+1/2} - \sigma_n\right) - \left(\hat{\dot{\varepsilon}}^p_{n+1/2} - \dot{\varepsilon}^p_n\right)$.

Using equations (18) and (17) into (16) we have :

$\forall v \in \mathcal{U}^S(\Omega,0) \otimes \mathcal{U}^T(I), \quad \forall \bar{\sigma}^* \in \mathcal{F}(0)$,

$$\int_{I\times\Omega} [\mathbb{K}^{-1} : \Delta\dot{\sigma}_{n+1} + \mathbb{A} : \Delta\sigma_{n+1} - \bar{\Delta}_{n+1}] : \bar{\sigma}^* d\Omega dt = -\int_{I\times\Omega} \rho\Delta\dot{u}_{n+1} \cdot \ddot{v}\, d\Omega dt$$

by defining $\Delta u_{n+1} = \lambda\bar{u}$, $\Delta\sigma_{n+1} = \lambda\bar{\sigma}$, $\bar{\sigma}^* = \lambda\bar{\sigma}^*$ and $\bar{\Delta}_{n+1} = \bar{\Delta}$, we can write :

$\forall v \in \mathcal{U}^S(\Omega,0) \otimes \mathcal{U}^T(I), \quad \forall \bar{\sigma}^* \in \mathcal{F}(0)$,

$$\int_\Omega \left[< \mathbb{A}\lambda^2 > : \bar{\sigma} + < \dot{\lambda}\lambda > \mathbb{K}^{-1} : \bar{\sigma} - < \bar{\Delta}\lambda > \right] : \bar{\sigma}^* d\Omega = -\int_\Omega \rho < \ddot{\lambda}\dot{\lambda} > \bar{u} \cdot v d\Omega \tag{19}$$

with $< \cdot > = \int_{[0,T]} (\cdot) dt$.

From equation (19) we can define $\varepsilon(\bar{u}) = \Delta\varepsilon_{n+1} = \left[< \mathbb{A}\lambda^2 > : \bar{\sigma} + < \dot{\lambda}\lambda > \mathbb{K}^{-1} : \bar{\sigma} - < \bar{\Delta}\lambda > \right]$, where it can be arrange in the more condensed form as :

$$\varepsilon(\bar{u}) = W^{-1} : \bar{\sigma} - \bar{\delta} \tag{20}$$

with $W^{-1} = < \mathbb{A}\lambda^2 > + < \dot{\lambda}\lambda > \mathbb{K}^{-1}$ and $\bar{\delta} = < \bar{\Delta}\lambda >$. By doing this, we have :

$\forall v \in \mathcal{U}^S(\Omega,0) \otimes \mathcal{U}^T(I), \quad \forall \bar{\sigma}^* \in \mathcal{F}(0)$,

$$\int_\Omega \varepsilon(\bar{u}) : \bar{\sigma}^* d\Omega = -\int_\Omega \rho < \ddot{\lambda}\dot{\lambda} > \bar{u} \cdot v d\Omega \tag{21}$$

The static admissibility to zero of $\bar{\sigma}$ reads :

$$\int_\Omega \bar{\sigma} : \varepsilon(v) d\Omega = -\int_\Omega \rho < \ddot{\lambda}\dot{\lambda} > \bar{u} \cdot v d\Omega, \quad \forall v \in \mathcal{U}^S(\Omega,0) \tag{22}$$

that finally becomes using equation (20) :

$$\int_\Omega W : \varepsilon(\bar{u}) : \varepsilon(v) d\Omega = -\int_\Omega W : \bar{\delta} : \varepsilon(v) d\Omega - \int_\Omega \rho < \ddot{\lambda}\dot{\lambda} > \bar{u} \cdot v d\Omega, \quad \forall v \in \mathcal{U}^S(\Omega,0) \tag{23}$$

By discretizing the equation (23) we get the following linear equation in function of the nodal values of the total displacement correction Δu_{n+1} as :

$$\bar{\mathbf{W}}\Delta\mathbf{u}_{n+1}^{[i]} = -\bar{\mathbf{F}} - \left[\zeta < \dot{\lambda}\dot{\lambda} > \mathbf{D} + < \ddot{\lambda}\dot{\lambda} > \mathbf{M}\right] \Delta\mathbf{u}_{n+1}^{[i-1]} \tag{24}$$

with $\bar{\mathbf{W}} = \int_\Omega W : \varepsilon(\bar{u}) : \varepsilon(v) d\Omega$, $\bar{\mathbf{F}} = \int_\Omega W : \bar{\delta} : \varepsilon(v) d\Omega$, $\mathbf{D}$ and $\mathbf{M}$ the damping and mass matrices respectively calculated at the initial elastic solution step $\mathcal{S}_0$. As seen from equation (24) we considered a proportional damping term, the damping constant ζ is chosen to be 10^{-4} or 10^{-5}.

The last equation allows to calculate the correction Δu_{n+1} and by consequence the total deformation $\Delta\varepsilon_{n+1}$. Finally, the space functions of the corrections of stress and plastic deformation at iteration $n+1$ are computed by using (20) and (18) as follows :

$$\mathbb{C}_{n+1}(x) = \bar{\sigma} = W : (\varepsilon(\bar{u}) + \bar{\delta}) \tag{25}$$

$$\mathbb{E}^p_{n+1}(x) = \frac{1}{< \lambda\dot{\lambda} >} \left[\varepsilon(\bar{u}) - < \lambda\dot{\lambda} > \mathbb{K}^{-1} : W : (\varepsilon(\bar{u}) + \bar{\delta})\right] \tag{26}$$

It must be notice that the resolution is do it in a quasi-static framework but this time considering the inertial effects as an external loading. The index $[i]$ denote the number of the iteration for the calculation of the space corrections of the mode, the nodal displacement correction at iteration "i" depends on the displacement correction at iteration "$i-1$" due to the necessity of the information of the acceleration of the mode.

Time function determination over $I = [0, T]$:

Once the space functions known, they are used to calculate the time function associated to the correction $n+1$ by minimizing the search direction of descent :

$$\{\lambda_{n+1}\} = \arg\min \left\| \dot{\lambda}_{n+1}\mathbb{E}^p_{n+1} - \mathbb{A} : (\lambda_{n+1}\mathbb{C}_{n+1}) + \bar{\Delta}_{n+1} \right\|_2^2 \tag{27}$$

with $\|\cdot\| = \int_{I\times\Omega} (\cdot)\, d\Omega dt$.

At this stage Lagrange polynomials of order 2 are used in order to catch the second derivative of the time function.

2.4 Error estimation

At each calculation of the global corrections an indicator of the quality of the solution can be calculated as the distance between the local solution and the linear one

$$\eta_{LATIN} = \frac{\left\| \hat{\dot{\varepsilon}}^p - \dot{\varepsilon}^p \right\|}{\left\| \hat{\dot{\varepsilon}}^p + \dot{\varepsilon}^p \right\|} + \frac{\|\hat{\sigma} - \sigma\|}{\|\hat{\sigma} + \sigma\|} \tag{28}$$

with $\|\cdot\| = \int_{I\times\Omega} (\cdot)\, d\Omega dt$.

3 Numeric results

The following test consist on a plate held on both edges to which a cyclic and uniform vertical density force is applied on its free surface as seen in Figure 3.

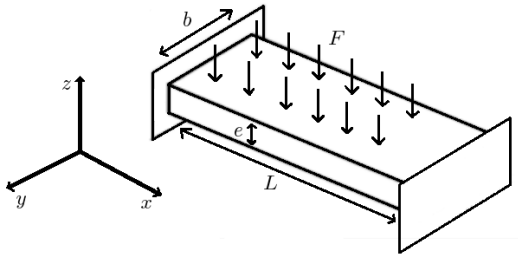


FIGURE 3 – Plate considered for the problem

A Kirchhoff model for the plate is considered. The constant associated to the perfect elastic-visco-plastic material are shown in Table 1.

TABLE 1 – Material properties.

Material properties	Value
$\tilde{n}$	2,5
K	1.220 [MPa $s^{1/\tilde{n}}$]
k	$K^{-\tilde{n}}$
Yield stress (σ_y)	80 [MPa]

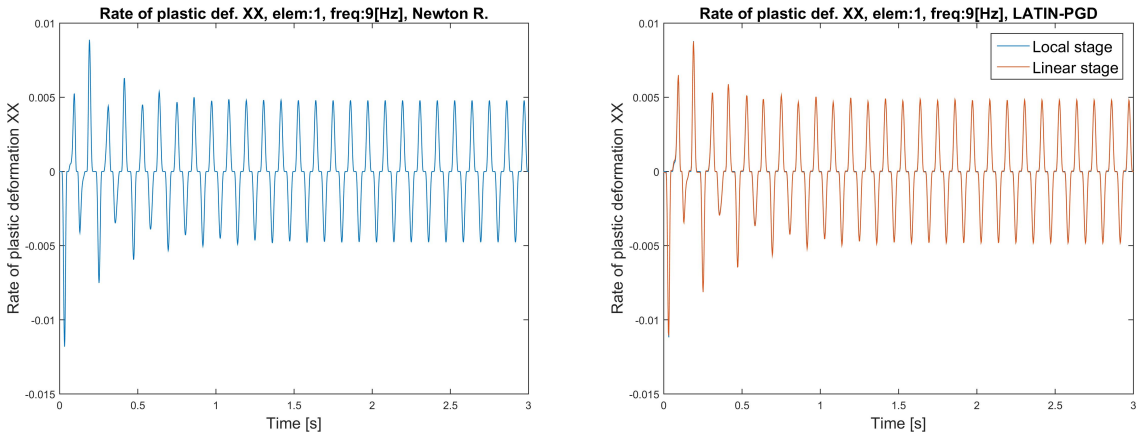
TABLE 2 – Geometrical considerations.

Geometric properties	Value
X dimension (L)	2 [m]
Y dimension (b)	0,4 [m]
Thickness (e)	0,02 [m]
N elements in X	10
N elements in Y	5
N elements in Z	3

The values associated to the geometrical considerations are presented on Table 2.

The material of the plate is considered to be steel with a Young modulus of 134 [GPa], a Poisson coefficient of 0,3 and a density of 7.165 [Kg/m^3]. With the geometric considerations, the model considered and the material properties the first natural frequency in flexion have a value of 22,6 [Hz], for this reason and in a way to slightly excite this vibrational mode, an external density force of 70.000 $\left[\frac{N}{m^2}\right]$ with an excitation frequency of 9 [Hz] is imposed. The total interval of time considered is $I = [0,3]$ seconds with 810 elements in time for the LATIN-PGD method (2430 temporal nodes), and 1620 elements for the Newton Raphson algorithm (1621 temporal nodes).

Figure 4 shows the evolution in time of the rate of plastic deformation for the most critical Gauss point in space (close to one fix side) and for the most critical component.



(a) $\dot{\varepsilon}^p_{xx}$ 1st space Gauss point, Newton Raphson.

(b) $\dot{\varepsilon}^p_{xx}$ 1st space Gauss point, LATIN-PGD.

FIGURE 4 – Newton Raphson vs Local and Global solutions of the LATIN-PGD solver for $\dot{\varepsilon}^p_{xx}$, 1st space Gauss point.

As seen from Figure 4, the transition zone obtained with the LATIN-PGD method have little amplitud differences compare with the one given by the Newton Raphson method, this can be explained due to the approximation of considering the inertial effects as an external excitation, despite this, the stationary zones is well reproduce and almost no difference between both models exist.

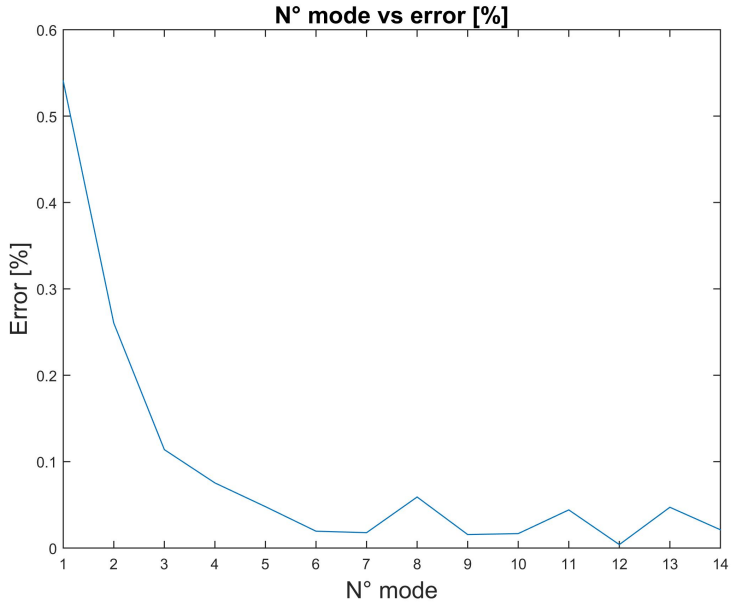


FIGURE 5 – Error of the LATIN-PGD method vs mode.

The preceding test was done by fixing the maximum number of modes for the calculation, for a fix number of 14 modes an error of 0,003 [%] is obtained by using the expression (28).

4 Conclusions

The test of the plate subjected to a cyclic vertical density force verifies the efficacy of the new approach of the LATIN-PGD for vibratoric dynamics problems, this points is of particular importance due to the assumption that the solution can be express as a sum of corrections of separable variables functions (Space and time). The present model naturally is not conceived where propagative solutions exist on the structure (fast dynamics), in this case the approximation of the corrections by a sum of separate variables functions don't work well and a big amount of modes should be calculated to represent the solution, a process that is computationally expensive and in those cases other types of methods for their resolution must be considered.

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