

Harmonic response Optimization using ESI TOPAZE

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Summary — Weight reduction is an important issue in the competitive industries, particularly in the industry of transport vehicles. To meet this objective, ESI Group has developed a disruptive and innovative optimization tool based on the technology of the level-set ([8], [9]). Unlike existing methods (power law, SIMP etc. [1], [4]), the level-set representation allows an accurate sharp knowledge of the boundary location. The linear static mechanics has already been discussed in previous edition of this congress ([5], [6]), so in this paper we focus on NVH problems such as modal search and harmonic response problems.

Keywords — Shape optimization, Topological optimization, Level-set, ESI TOPAZE, Harmonic response.

1. Introduction

Weight reduction is a major issue for the industry, especially in vehicles industry where light vehicles are developed to reduce environmental impacts. The first goal of the topological optimization is to find better shapes that minimize an objective, most of the time the volume or the mass, but preserve enough level of performance. In previous papers in CSMA conference ([5], [6]) and previously in [2] we described the process of optimization for static linear problems. In this paper we focus on the formulation for modal search and harmonic response mechanic.

2. Level-set representation

2.1. Shape representation

Most of existing optimization tools use a density method to represent the shape on a given mesh. In our tool, we chose to use the now well-known level-set method for the shape representation.

Let $D \in \mathbb{R}^3$ be an open bounded we call ‘Design Space’. It is the maximum space where to search for an optimal shape. Let Ω be an admissible shape, which then verifies $\Omega \subset D$.

To represent the shape Ω in the Design Space, we define a function ψ (the level-set function) on D such as:

$$\begin{cases} \psi(x) = 0 \Leftrightarrow x \in \partial\Omega, \\ \psi(x) < 0 \Leftrightarrow x \in \Omega, \\ \psi(x) > 0 \text{ otherwise.} \end{cases} \quad (1)$$

With these notations, we have access to some interesting geometrical properties: the local normal $n = \frac{\nabla\psi}{\|\nabla\psi\|}$ and the local mean curvature $\kappa = \nabla \cdot n$.

A common choice for this function is the signed distance to the boundary. This kind of choice for level-set function implies a strong regularity of the function and $\|\nabla\psi\| = 1$. Also, it helps for geometrical criteria such as thickness.

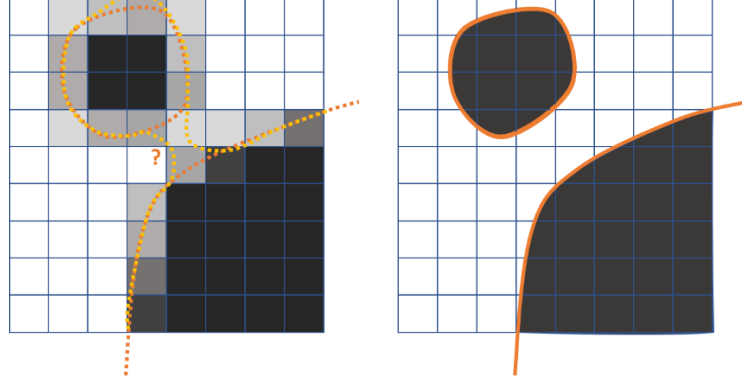


Figure 1 - Fuzzy boundary representation with cell density showing uncertainty (left) versus sharp representation of the boundary with level-set (right)

The evolution of the shape is governed by the now well-known Hamilton Jacobi problem:

$$\frac{\partial\psi}{\partial t} - v|\nabla\psi| = 0. \quad (2)$$

In this equation, t is a fictitious time representing the optimization step increment. v is called ‘descent direction’ and is computed by the optimizer.

2.2. Computation of the descent direction

The main goal of the optimizer is to compute the descent direction. It has been extensively studied ([7], [1]). With the standard methods, a lot of algorithms are available (SLP, MMA, SQP, MFD, Uzawa, etc.) but all these methods relies on explicit optimization parameters. With the use of level-set, the optimization parameter is implicit (the place where the level-set function is null).

The optimization problem is written as:

$$\begin{cases} \min_{\Omega} f(\Omega), \\ g_i(\Omega) \leq 0 \forall i, \end{cases} \quad (3)$$

where f is the objective function and g_i the constraints. Both objective and constraints are called ‘criteria’ of the optimization.

3. Application to modal search analysis

In the following, we split $\partial\Omega = \Gamma$ into two parts: Γ_D the Dirichlet part and Γ_N the Neumann part.

Search for Eigenmodes of a mechanical system consists on solving the following problem:

$$\begin{cases} -\nabla \cdot \sigma_u = \omega^2 \rho u & \text{in } \Omega, \\ u = 0 & \text{on } \Gamma_D, \\ \sigma_u \cdot n = 0 & \text{on } \Gamma_N, \end{cases} \quad (4)$$

where, σ_u is the Cauchy’s stress tensor, ρ the material density.

It is a well-known problem that admits a countable family of positive Eigenvalues, sorted as:

$$0 \leq \omega_1^2 \leq \omega_2^2 \leq \dots \leq \omega_k^2 \rightarrow +\infty. \quad (5)$$

Then the corresponding u_k is called the k-th Eigenvector.

With this type of analysis, we usually control the values of Eigenfrequencies. We write the criterion as:

$$J(\Omega) = \frac{\omega_k}{2\pi}. \quad (6)$$

This problem is self-adjoint (see [3] for the demonstration). The gradient of $J(\Omega)$ is:

$$J'(\Omega)(\theta) = \frac{\int_{\Gamma} \theta \cdot n (\sigma_{u_k} : \varepsilon_{u_k} - \omega_k^2 \rho u_k^2)}{4\pi\omega_k \int_{\Omega} \rho u_k^2}. \quad (7)$$

4. Application to harmonic response

4.1. The harmonic response problem

The harmonic response problem is a mechanical problem where the solicitations are in a sinusoidal form. In such problem, velocity and acceleration terms of the equilibrium equation are considered.

The problem is expressed as follow:

$$\begin{cases} \rho \ddot{u}_{\tilde{\omega}} + c \dot{u}_{\tilde{\omega}} - \nabla \cdot \sigma_{u_{\tilde{\omega}}} = f_{\tilde{\omega}} & \text{in } \Omega, \\ u_{\tilde{\omega}} = g_{\tilde{\omega}} & \text{on } \Gamma_D, \\ \sigma_{u_{\tilde{\omega}}} \cdot n = h_{\tilde{\omega}} & \text{on } \Gamma_N. \end{cases} \quad (8)$$

To ease notations, we will use the complex form, with $i^2 = -1$.

$$f_{\tilde{\omega}} = \text{Re}(f e^{i\tilde{\omega}t}) = \text{Re}((f_{re} + i f_{im}) e^{i\tilde{\omega}t}) = f_{re} \cos \tilde{\omega}t + f_{im} \sin \tilde{\omega}t.$$

We use the same notation for $u_{\tilde{\omega}}$, $g_{\tilde{\omega}}$ and $h_{\tilde{\omega}}$.

Then we deduce the velocity:

$$\begin{aligned} \dot{u}_{\tilde{\omega}} &= \frac{\partial u_{\tilde{\omega}}}{\partial t} = \text{Re}(i\tilde{\omega}(u_{re} + i u_{im}) e^{i\tilde{\omega}t}) = \tilde{\omega} \text{Im}((u_{re} + i u_{im}) e^{i\tilde{\omega}t}) \\ &= -\tilde{\omega} u_{im} \cos \tilde{\omega}t + \tilde{\omega} u_{re} \sin \tilde{\omega}t. \end{aligned}$$

And the acceleration:

$$\ddot{u}_{\tilde{\omega}} = \frac{\partial^2 u_{\tilde{\omega}}}{\partial t^2} = \text{Re}(-\tilde{\omega}^2 (u_{re} + i u_{im}) e^{i\tilde{\omega}t}) = -\tilde{\omega}^2 u_{\tilde{\omega}} = -\tilde{\omega}^2 u_{re} \cos \tilde{\omega}t + \tilde{\omega}^2 u_{im} \sin \tilde{\omega}t.$$

By replacing in (8), we can divide by $e^{i\tilde{\omega}t}$:

$$\text{Re} \left(\begin{cases} -\rho \tilde{\omega}^2 u + i c \tilde{\omega} u - \nabla \cdot \sigma_u = f & \text{in } \Omega \\ u = g & \text{on } \Gamma_D \\ \sigma_u \cdot n = h & \text{on } \Gamma_N \end{cases} \right). \quad (9)$$

This can be expressed in a matrix form:

$$(-\tilde{\omega}^2 M + i\tilde{\omega}C + K)U_{\tilde{\omega}} = \tilde{F}_{\tilde{\omega}}. \quad (10)$$

Two methods are possible for solving this problem: the direct and the modal methods.

4.2. Criterion definition

In general case, the problem (10) needs to be solved for a range of excitation frequency $\tilde{\omega} = [\tilde{\omega}_{low}, \tilde{\omega}_{high}]$.

Like in static problem, the general form of criterion for a given excitation frequency $\tilde{\omega}$ is:

$$G_{\tilde{\omega}}(\Omega) = \left(\int_{\Omega} j(u_{\tilde{\omega}}) \right)^{\beta} + \left(\int_{\partial\Omega} l(u_{\tilde{\omega}}) \right)^{\gamma}, \quad (11)$$

with $\beta \geq 1$, $\alpha \geq 1$, j and l are derivable functions on Ω and $\partial\Omega$ respectively.

For a given frequency composition derivable function f_{fc} , the criterion is then

$$J(\Omega) = f_{fc}(G_{\tilde{\omega}}(\Omega)). \quad (12)$$

The integral on the frequency range is a simple example of the function f_{fc} . Another example, the dynamic stiffness is presented in the results section.

The general form for the gradient is then:

$$G'_{\tilde{\omega}}(\Omega)(\theta) = \int_{\partial\Omega} \left(C_{j0}j - \rho\tilde{\omega}^2 Re(u_{\tilde{\omega}} \cdot p_{\tilde{\omega}}^*) + c\tilde{\omega}Im(u_{\tilde{\omega}} \cdot p_{\tilde{\omega}}^*) - Re(f \cdot p_{\tilde{\omega}}^*) + Re(\sigma_{p_{\tilde{\omega}}^*}: \varepsilon_{u_{\tilde{\omega}}}) \right. \\ \left. + C_{l0} \left(\frac{\partial l}{\partial n} + \kappa l \right) \right) (\theta \cdot n) - \int_{\Gamma_n} \left(\frac{\partial Re(p_{\tilde{\omega}}^* \cdot g)}{\partial n} + \kappa Re(p_{\tilde{\omega}}^* \cdot g) \right) (\theta \cdot n), \quad (13)$$

with $C_{j0} = \beta \left(\int_{\Omega} j \right)^{\beta-1}$ and $C_{l0} = \gamma \left(\int_{\partial\Omega} l \right)^{\gamma-1}$.

$p_{\tilde{\omega}}$ is the solution of the following adjoint problem, where $p_{\tilde{\omega}}^*$ is the complex conjugate of $p_{\tilde{\omega}}$:

$$\begin{cases} -\rho\tilde{\omega}^2 p_{\tilde{\omega}}^* + ic\tilde{\omega}p_{\tilde{\omega}}^* - \nabla \cdot \sigma_{p_{\tilde{\omega}}^*} = -C_{j0}j'^* & \text{in } \Omega, \\ p_{\tilde{\omega}}^* = -C_{l0}l'^* & \text{on } \Gamma_D, \\ \sigma_{p_{\tilde{\omega}}^*} \cdot n = -C_{l0}l'^* & \text{on } \Gamma_N. \end{cases} \quad (14)$$

This adjoint problem has the same form as the direct mechanical problem. So, the same method can be used to solve this problem. Most of the time, the direct and adjoint solution are solved together to spare computation time.

4.3. Adjoint computation using direct method

For the direct method the adjoint is computed using the matrix defined at (10), used for mechanical direct problem with a different right-hand side member according to (8).

$$(-\tilde{\omega}^2 M + i\tilde{\omega}C + K)P_{\tilde{\omega}} = \tilde{R}_{\tilde{\omega}}. \quad (15)$$

The main drawback of this method is that, even for mechanics alone, it requires a matrix inversion for each frequency sample. This can be very time-consuming when the number of sample increases.

4.4. Adjoint computation using modal method

For the modal method we first solve a modal search problem as described in (4) to get a modal basis of vectors. We truncate this basis to M vectors: $\Phi = (\phi_1, \dots, \phi_M)$.

We normalize each vector by the mass matrix:

$$\forall k, \phi_k^T M \phi_k = 1. \quad (16)$$

Let $Q_{\tilde{\omega}} = (Q_k^{\tilde{\omega}})$ be the projection of $U_{\tilde{\omega}}$ in the modal base Φ . The system (10) is then rewritten as:

$$(-\tilde{\omega}^2 [\Phi^T M \Phi] + i\tilde{\omega} [\Phi^T C \Phi] + [\Phi^T K \Phi]) Q_{\tilde{\omega}} = \Phi^T \tilde{F}_{\tilde{\omega}}. \quad (17)$$

By the properties of Eigenmodes, both $[\Phi^T M \Phi]$ and $[\Phi^T K \Phi]$ are diagonal with respective diagonal coefficients m_k and k_k . The damping can be chosen to also be diagonal with diagonal coefficient c_k .

Then the problem (17) consists of M independent problems where the excitation frequency is only a factor without the need of a matrix factorization. Then both direct and adjoint solution can be easily computed.

5. Results

5.1. 2D cantilever with Dynamic stiffness

For an excitation's range problem $[\omega_{low}, \omega_{high}]$, we can compare the static computation of the compliance and the harmonic response version.

We define the dynamic stiffness as:

$$J(\Omega) = 10^{-(\omega_{high}-\omega_{low}) \int_{\omega_{low}}^{\omega_{high}} \log_{10}(G_m(\Omega))}, \quad (18)$$

in which:

$$G_m(\Omega) = \left(\int_{\Omega} k(x) \cdot |U_{\tilde{\omega}_m}(x)|^\alpha dx \right)^{\frac{1}{\alpha}}. \quad (19)$$

On a 2D cantilever, the figure 2 shows the comparison between static-only results and with dynamic stiffness applied.

5.2. 3D cantilever with Dynamic compliance

For another example, the criterion used the dynamic compliance by replacing $G_m(\Omega)$:

$$G_m(\Omega) = \int_{\Omega} \sigma_{u_{\tilde{\omega}}} : \varepsilon_{u_{\tilde{\omega}}}. \quad (20)$$

It is possible to compare the different methods used to obtain a comparable shape. (see Figure 3)

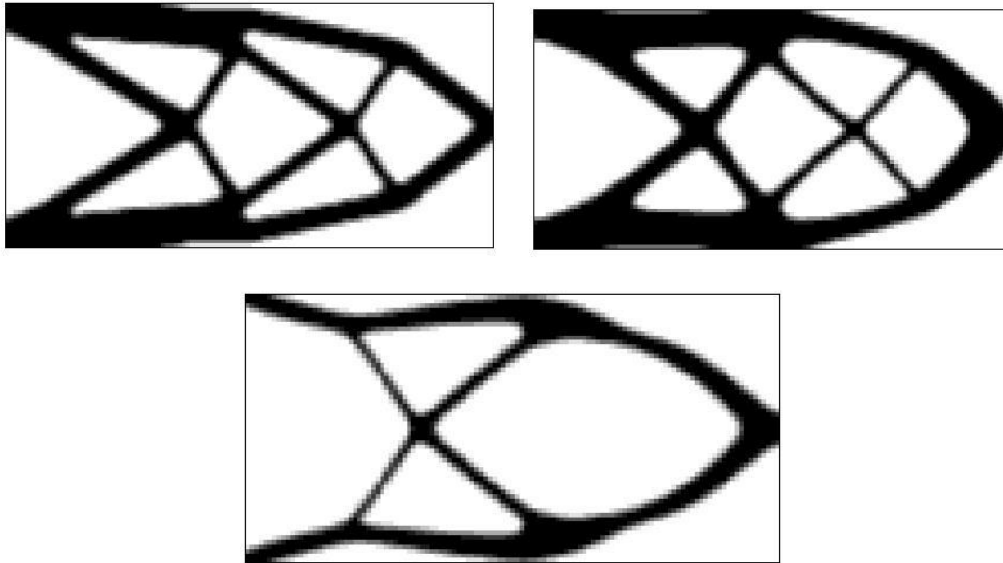


Figure 2 – Results for static compliance <80 only (top left), for static compliance <80 and dynamic stiffness > 0.1 (top right), for static compliance <220 and dynamic stiffness > 0.1 (bottom)

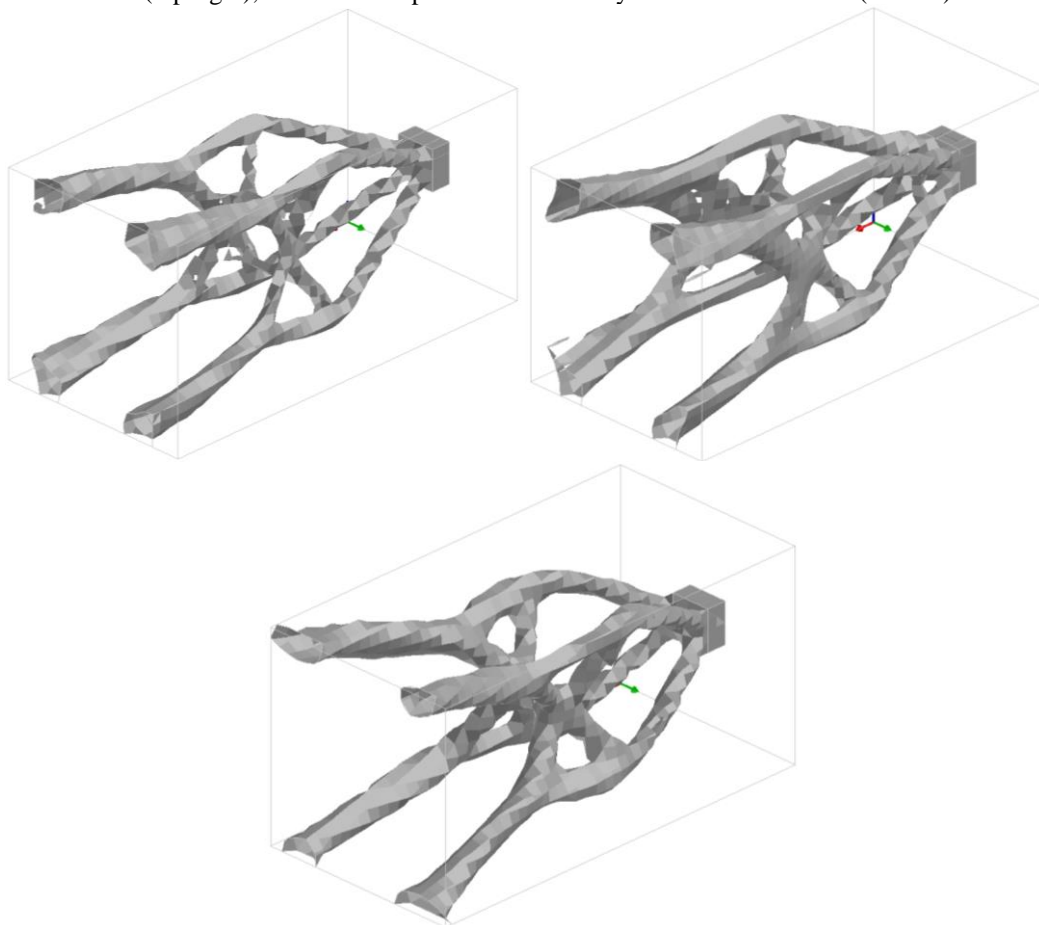


Figure 3 – Result for static compliance $<20k$ only (top left), for dynamic compliance $<20k$ with direct method (top right), for dynamic compliance $<20k$ with modal method(bottom)

As expected, with enough modes in the modal base, the modal method gives the same results as the direct method. Unfortunately, the right number of modes depends on the test case and need an expert eye to be setup.

6. Conclusion

We successfully set up the harmonic response physics into ESI TOPAZE with both direct and modal methods. These criteria and their adjoints are like the static ones. Thus, large part of the theory and methods still apply. From software development point of view, the computation of the adjoint solution is mainly built using parts available within the direct solver. With this development, we can now deal with sophisticated criteria, which consider both phase and amplitude in a large range of frequencies.

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